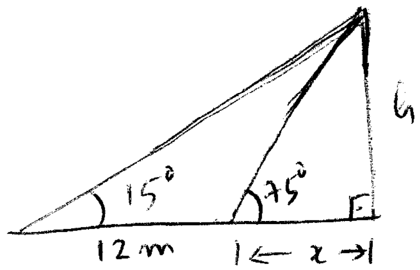
GABARITO TRIGONOMETRIA:

01)



$h = ?$

$\tan 75° = \dfrac{h}{x} \Rightarrow h = \tan 75° \cdot x$;

onde $\tan 75° = \tan(30° + 45°)$

$$= \frac{\tan 30° + \tan 45°}{1 - \tan 30° \cdot \tan 45°} = \frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3} \cdot 1}$$

$$= \frac{3+\sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{3-\sqrt{3}} = \frac{3+\sqrt{3}}{3-\sqrt{3}} \cdot \frac{3+\sqrt{3}}{3+\sqrt{3}}$$

$$= \frac{9 + 6\sqrt{3} + 3}{9-3} = 2 + \sqrt{3}; \text{ e daí:}$$

$\boxed{h = (2+\sqrt{3}) \cdot x}$ (*)

$\tan 15° = \dfrac{h}{12 + x}$; onde $\tan 15° = \tan(45° - 30°) =$

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \cdot \tan 30°} = \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1 \cdot \frac{\sqrt{3}}{3}}$$

$$= \frac{3-\sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{3+\sqrt{3}} = \frac{3-\sqrt{3}}{3+\sqrt{3}} \cdot \frac{3-\sqrt{3}}{3-\sqrt{3}}$$

$$= \frac{9 - 6\sqrt{3} + 3}{9-3} = 2 - \sqrt{3}; \text{ e daí:}$$

$2 - \sqrt{3} = \dfrac{h}{12+x} \Rightarrow (2-\sqrt{3})(12+x) = h = (2+\sqrt{3})x$

por (*)

$$24 + \cancel{2x} - 12\sqrt{3} - \sqrt{3}x = \cancel{2x} + \sqrt{3}x$$

$$24 - 12\sqrt{3} = 2\sqrt{3}x$$

$$\Rightarrow x = \frac{24 - 12\sqrt{3}}{2\sqrt{3}} = \frac{12 - 6\sqrt{3}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow x = \frac{(12 - 6\sqrt{3}) \cdot \sqrt{3}}{3} = (4 - 2\sqrt{3}) \cdot \sqrt{3}$$

$$\Rightarrow x = 4\sqrt{3} - 6 \quad \text{ou} \quad 2(2 - \sqrt{3}) \cdot \sqrt{3}\ m.$$

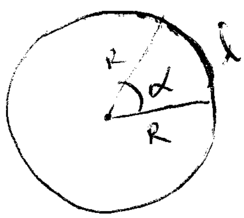
Assim, voltando em (*), temos:

$$h = (2 + \sqrt{3})x = (2 + \sqrt{3}) \cdot 2(2 - \sqrt{3}) \cdot \sqrt{3}\ m$$

$$= 2\sqrt{3} \cdot (4 - 3) = 2\sqrt{3}\ m$$

QUESTÃO 02:

a)



$R = 36\ cm$

$l = 3\pi\ cm.$

$$\alpha_{(rad)} = \frac{l}{R} = \frac{3\pi}{36} = \frac{\pi}{12}\ rad$$

$$\alpha_{(graus)} = \frac{180^\circ}{12} = 15^\circ$$

b) 360° —— $\pi \cdot (36)^2$

15° —— A

$$360 \cdot A = 15 \cdot (36)^2 \cdot \pi$$

$$\Rightarrow A = \frac{15 \cdot (36)^2 \cdot \pi}{\cancel{36} \cdot 10} = \frac{3 \cdot 36\pi}{2} = 54\pi\ cm^2$$

c) $\cot\alpha = \cot 15^\circ$

Pelo item anterior,

$$\cot 15^\circ = \tan 75^\circ = 2+\sqrt{3}.$$

QUESTÃO 03: Temos:

- $\text{sen}(\underbrace{90^\circ+x}_{\in 2^\underline{o}q}) = \text{sen}(180^\circ-(90^\circ+x)) = \text{sen}(90^\circ-x) = \cos x.$

- $\tan(\underbrace{270^\circ+x}_{\in 4^\underline{o}q}) = -\tan(360^\circ-(270^\circ+x)) = -\tan(90^\circ-x)$
  $= -\cot x.$

- $\cos(\underbrace{720^\circ-x}_{\in 4^\underline{o}q}) = +\cos(360^\circ-(720^\circ-x)) =$
  $= \cos(\underbrace{360^\circ+x}_{\in 1^\underline{o}q}) = \cos x.$

- $\csc(\underbrace{180^\circ+x}_{\in 3^\underline{o}q}) = -\csc(180^\circ+x-180^\circ) = -\csc x.$

Assim:

$$y = \frac{\cancel{\cos x}\cdot(-\cot x)}{\cancel{\cos x}\cdot(-\csc x)} = \frac{\frac{\cos x}{\text{sen}x}}{\frac{1}{\text{sen}x}} = \frac{\cos x}{\cancel{\text{sen}x}}\cdot\frac{\cancel{\text{sen}x}}{1}$$

$$= \cos x.$$

QUESTÃO 04: $x+y=210°$; $\begin{cases} x \in 1.°q \\ y \in 2.°q \end{cases}$

$\tan y = -\frac{\sqrt{5}}{2}$.

Achar: $\csc x$.

Note que $x = 210° - y$ e daí:

$$\operatorname{sen} x = \operatorname{sen}(210° - y) = \operatorname{sen}210° \cdot \cos y - \operatorname{sen} y \cdot \cos 210°$$

$$= -\operatorname{sen}30° \cdot \cos y - \operatorname{sen} y \cdot (-\cos 30°)$$

$$= -\frac{1}{2} \cdot \cos y + \frac{\sqrt{3}}{2} \cdot \operatorname{sen} y$$

$$1 + \tan^2 y = \sec^2 y \implies \sec y = -\sqrt{1 + \left(-\frac{\sqrt{5}}{2}\right)^2}$$

$$= -\sqrt{1 + \frac{5}{4}} = -\sqrt{\frac{4+5}{4}}$$

$$= -\frac{3}{2}.$$

$\implies \cos y = -\frac{2}{3}$. e daí: $\operatorname{sen} y = +\sqrt{1 - \left(-\frac{2}{3}\right)^2}$

$$= \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$$

Portanto:

$$\operatorname{sen} x = -\frac{1}{2} \cdot \cos y + \frac{\sqrt{3}}{2} \operatorname{sen} y =$$

$$= -\frac{1}{2} \cdot \left(-\frac{2}{3}\right) + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{5}}{3} = \frac{2 + \sqrt{15}}{6}$$; e então

$$\csc x = \frac{1}{\ldots} = \frac{6}{2 + \sqrt{15}}$$

QUESTÃO 05:

(a) $$\frac{\operatorname{sen}\alpha+\operatorname{sen}\beta}{\cos\alpha+\cos\beta}=\frac{\cancel{2}\cdot\operatorname{sen}\frac{\alpha+\beta}{2}\cdot\cancel{\cos\frac{\alpha-\beta}{2}}}{\cancel{2}\cos\frac{\alpha+\beta}{2}\cdot\cancel{\cos\frac{\alpha-\beta}{2}}}=\tan\frac{\alpha+\beta}{2}.$$

(b) $$3-4\cos 2\alpha+\cos 4\alpha=3-4(\cos^2\alpha-\operatorname{sen}^2\alpha)+\cos^2 2\alpha-\operatorname{sen}^2 2\alpha$$

$$=3-4(1-\operatorname{sen}^2\alpha-\operatorname{sen}^2\alpha)+(\cos^2\alpha-\operatorname{sen}^2\alpha)^2-(2\operatorname{sen}\alpha\cos\alpha)^2$$

$$=3-4+8\operatorname{sen}^2\alpha+(1-2\operatorname{sen}^2\alpha)^2-4\operatorname{sen}^2\alpha(1-\operatorname{sen}^2\alpha)$$

$$=-1+8\operatorname{sen}^2\alpha+1-4\operatorname{sen}^2\alpha+4\operatorname{sen}^4\alpha-4\operatorname{sen}^2\alpha+4\operatorname{sen}^4\alpha$$

$$=8\cdot\operatorname{sen}^4\alpha.$$

QUESTÃO 06:

(a) $$\cos 15^\circ=\cos(45^\circ-30^\circ)=\cos 45^\circ\cdot\cos 30^\circ+\operatorname{sen}45^\circ\cdot\operatorname{sen}30^\circ$$

$$=\frac{\sqrt2}{2}\cdot\frac{\sqrt3}{2}+\frac{\sqrt2}{2}\cdot\frac{1}{2}=\frac{\sqrt6+\sqrt2}{4}>0.$$

(b) $$\cos 15^\circ=\cos\left(\frac{30^\circ}{2}\right)=+\sqrt{\frac{1+\cos 30^\circ}{2}}=$$

$$=\sqrt{\frac{1+\frac{\sqrt3}{2}}{2}}=\frac{\sqrt{2+\sqrt3}}{2}>0.$$

De fato, chame $x=\frac{\sqrt6+\sqrt2}{4}>0$ e $y=\frac{\sqrt{2+\sqrt3}}{2}>0$.

Então: $$x^2=\frac{6+2\sqrt{12}+2}{16}=\frac{8+4\sqrt3}{16}=\frac{2+\sqrt3}{4};\ \text{e}$$

$$y^2=\left(\frac{\sqrt{2+\sqrt3}}{2}\right)^2=\frac{2+\sqrt3}{4}$$

Assim, $x^2=y^2\Rightarrow x=-y$ ou $x=y$.

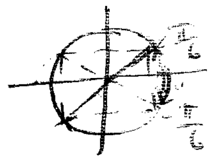
Como $y, x, \geq 0$, segue que $x = y$, como queríamos.

QUESTÃO 07:

(a) $2 \cdot \tan^2 x + \sec^2 x = 2$

$2 \tan^2 x + 1 + \tan^2 x - 2 = 0$

$3 \tan^2 x = 1$



$\tan^2 x = \frac{1}{3} \Rightarrow \tan x = \pm \frac{\sqrt{3}}{3}$

$\tan x = + \frac{\sqrt{3}}{3} \Leftrightarrow x = k\pi + \frac{\pi}{6}, \; k \in \mathbb{Z}$

$\tan x = - \frac{\sqrt{3}}{3} \Leftrightarrow x = k\pi - \frac{\pi}{6}, \; k \in \mathbb{Z}$

$S = \{ k\pi \pm \frac{\pi}{6} ; \; k \in \mathbb{Z} \}$.

(b) $\operatorname{sen} 2x + \cos x = 0$

$2 \cdot \operatorname{sen} x \cos x + \cos x = 0$

$\cos x (2 \cdot \operatorname{sen} x + 1) = 0$

- $\cos x = 0$  $\Leftrightarrow x = k\pi + \frac{\pi}{2}$.

- $2 \operatorname{sen} x + 1 = 0 \Leftrightarrow \operatorname{sen} x = -\frac{1}{2}$ (1ª q seria: $x = \frac{\pi}{6}$)

$\Leftrightarrow x = 2k\pi + \left(\pi + \frac{\pi}{6}\right) = 2k\pi + \frac{7\pi}{6}$

ou

$x = 2k\pi - \frac{\pi}{6}$.

SOLUÇÃO: $\{ k\pi + \frac{\pi}{2} ; \; 2k\pi + \frac{7\pi}{6} ; \; 2k\pi - \frac{\pi}{6} ; \; k \in \mathbb{Z} \}$.