TRIGONOMETRIA. LISTA DE EXERCÍCIOS 02.

SOLUÇÃO DE ALGUNS EXERCÍCIOS.

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01) (a) $\overset{\frown}{AM} = 1290^\circ$

$$\begin{array}{r|l} 1290^\circ & 360^\circ \\ \hline -1080^\circ & 3 \\ \hline 210^\circ & \end{array}$$

- menor determinação: $\alpha = 210^\circ$
- o arco dá 3 voltas no ciclo.
- EXPR. GERAL:

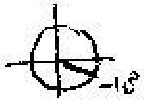
$$\overset{\frown}{AM}_k = k.360^\circ + 210^\circ.$$

(c) $\overset{\frown}{AP} = -2170^\circ$

$$\begin{array}{r|l} -2170^\circ & 360^\circ \\ \hline +2160^\circ & -6 \\ \hline -10^\circ & \end{array}$$

Como a menor determinação α deve ser o menor arco positivo côngruo a $\overset{\frown}{AP}$, temos:

$$\alpha = -10^\circ + 360^\circ = 350^\circ$$



Assim, temos que:

- menor determ.: $\alpha = 350^\circ$
- o arco dá -6 voltas no ciclo (i.e, 6 voltas no sentido horário)
- EXPR. GERAL:

$$\overset{\frown}{AP}_k = k.360^\circ + 350^\circ.$$

02) (a)

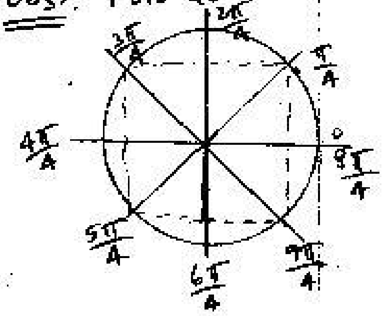
$$y = \frac{4.\cos\frac{\pi}{3} + 2.\operatorname{sen}\frac{\pi}{4}}{\tan^2\frac{\pi}{6} - 1} = \frac{4.\left(\frac{1}{2}\right) + 2.\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{3}}{3}\right)^2 - 1} =$$

$$= \frac{2 + \sqrt{2}}{\frac{3}{9} - 1} = \frac{2 + \sqrt{2}}{\frac{1}{3} - \frac{3}{3}} = \frac{2 + \sqrt{2}}{-\frac{2}{3}} =$$

$$= -\frac{3}{2}.(2 + \sqrt{2})$$

$$\text{(b)}\quad y = \frac{\operatorname{sen}\frac{7\pi}{4} + \tan\frac{3\pi}{4}}{2\cdot\cos\frac{\pi}{3} - \tan 6\pi}$$

Obs.: NOTE QUE:



$$\begin{cases} \operatorname{sen}\frac{7\pi}{4} = -\operatorname{sen}\frac{\pi}{4} = -\frac{\sqrt{2}}{2}. \\ \tan\frac{3\pi}{4} = -\tan\frac{\pi}{4} = -1 \end{cases}$$

Assim:

$$y = \frac{-\frac{\sqrt{2}}{2} - 1}{2\cdot\frac{1}{2} - 0} = -\frac{\sqrt{2}}{2} - 1 = -\frac{2+\sqrt{2}}{2}.$$

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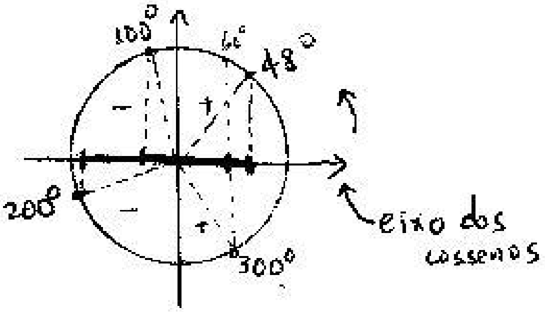
03) (b)



Olhando as projeções dos lados terminais dos arcos no eixo dos cossenos, percebemos que

$$\cos 200^\circ < \cos 100^\circ < \cos 300^\circ < \cos 48^\circ.$$

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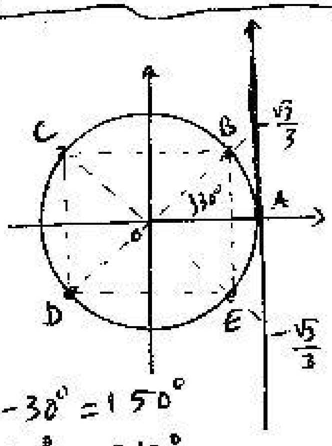
04)



$\operatorname{sen}\widehat{AB} = \operatorname{sen}30^\circ = \frac{1}{2}$

$\cos\widehat{AB} = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$\tan\widehat{AB} = \tan 30^\circ = \frac{\sqrt{3}}{3}.$

$\operatorname{sen}\widehat{AC} = \operatorname{sen}150^\circ = \frac{1}{2}$

$\cos\widehat{AC} = \cos 150^\circ = -\frac{\sqrt{3}}{2}$

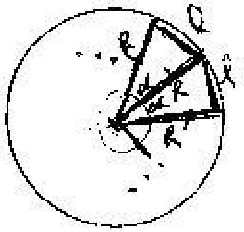
$\tan\widehat{AC} = \tan 150^\circ = -\frac{\sqrt{3}}{3}$

$\widehat{AB} = 30^\circ$

$\widehat{AC} = 180^\circ - 30^\circ = 150^\circ$

05)

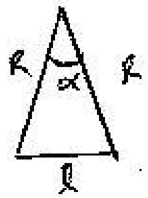
(a)



São n triângulos isósceles. Então, temos que o ângulo α de cada vértice (com o centro da circunferência) será $\alpha = \frac{2\pi}{n}$.

(b)

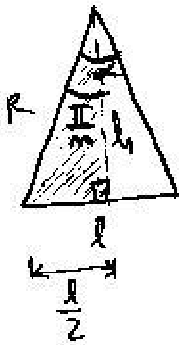
$\alpha = \frac{2\pi}{n}$



O polígono regular é formado por n triângulos isosceles iguais, c.f. o desenhado ao lado. Assim; sendo A_n a área do referido polígono, temos:

$$A_n = 6 . A_3$$

, onde A_3 é a área de cada um dos n triângulos isósceles.



Note que:

$$\tan \frac{\pi}{n} = \frac{\frac{l}{2}}{h} \Rightarrow h = \frac{l}{2 . \tan \frac{\pi}{n}},$$

e daí:

$$A_3 = \frac{b . h}{2} = \frac{l . \frac{l}{2 \tan \frac{\pi}{n}}}{2} = \frac{l^2}{4 . \tan \frac{\pi}{n}},$$

e portanto,

$$A_n = n . A_3 = \frac{n . l^2}{4 . \tan \frac{\pi}{n}}.$$

(c)

A_4 : ÁREA DO QUADRADO:

$$A_4 = \frac{4 . l^2}{4 . \tan \frac{\pi}{4}} = l^2$$

A_3 : ÁREA DO TRIÂNGULO :

$$A_3 = \frac{3 \cdot \ell^2}{4 \cdot \tan\frac{\pi}{3}} = \frac{3\ell^2}{4 \cdot \sqrt{3}} = \frac{3\ell^2}{4\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\ell^2\sqrt{3}}{4}.$$

A_6 : ÁREA DO HEXÁGONO REGULAR :

$$A_6 = \frac{6 \cdot \ell^2}{4 \cdot \tan\frac{\pi}{6}} = \frac{3\ell^2}{2 \cdot \frac{\sqrt{3}}{3}} = \frac{3\ell^2}{2 \cdot \frac{\sqrt{3}}{3} \cdot \frac{\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{3\ell^2}{2 \cdot \frac{1}{\sqrt{3}}} = \frac{3\sqrt{3}\ell^2}{2}$$

(d) A_5 : ÁREA DO PENTÁGONO REGULAR :

$$\cos 36^\circ = \frac{\varphi}{2}$$

$$A_5 = \frac{5 \cdot \ell^2}{4 \cdot \tan\frac{\pi}{5}} \quad ; \quad \tan\frac{\pi}{5} = \tan 36^\circ$$

$$1 + \tan^2 36^\circ = \sec^2 36^\circ$$

$$\tan 36^\circ = \sqrt{\sec^2 36^\circ - 1} \; ;$$

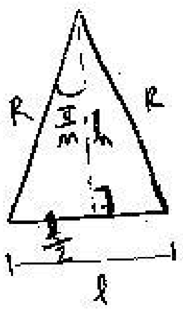
e como $\sec 36^\circ = \dfrac{1}{\cos 36^\circ} = \dfrac{2}{\varphi}$; vem que

$$\tan 36^\circ = \sqrt{\left(\frac{2}{\varphi}\right)^2 - 1} = \sqrt{\frac{4-\varphi^2}{\varphi^2}} = \frac{\sqrt{4-\varphi^2}}{\varphi}$$

e daí :

$$A_5 = \frac{5 \cdot \ell^2}{4 \cdot \frac{\sqrt{4-\varphi^2}}{\varphi}} = \frac{5\varphi \cdot \ell^2}{4\sqrt{4-\varphi^2}}$$

06) $A_n = ?$



Seja A_3 a área de cada triângulo isósceles. Note que:

- $\sin\frac{\pi}{n} = \frac{\frac{l}{2}}{R} \Rightarrow l = 2R \cdot \sin\frac{\pi}{n}$.
- $\cos\frac{\pi}{n} = \frac{h}{R} \Rightarrow h = R \cdot \cos\frac{\pi}{n}$

Assim:

$$A_n = n \cdot \frac{b \cdot h}{2} = \frac{n}{2} \cdot b \cdot h =$$

$$= \frac{n}{\cancel{2}} \cdot \cancel{2}R \cdot \sin\frac{\pi}{n} \cdot R \cdot \cos\frac{\pi}{n}$$

$$\Rightarrow \boxed{A_n = n \cdot R^2 \cdot \sin\frac{\pi}{n} \cdot \cos\frac{\pi}{n}}$$

$$\lim_{n \to +\infty} A_n = \lim_{n \to +\infty} \overbrace{n\, R^2 \cdot \sin\frac{\pi}{n} \cdot \cos\frac{\pi}{n}}^{\text{“}\infty \cdot 0\text{”}} =$$

$$= \lim_{n \to +\infty} \frac{R^2 \cdot \sin\frac{\pi}{n} \cdot \cos\frac{\pi}{n} \cdot \pi}{\frac{1}{n} \cdot \pi}$$

$$= \lim_{n \to +\infty} \pi R^2 \cdot \underbrace{\frac{\sin\frac{\pi}{n}}{\frac{\pi}{n}}}_{\to 1} \cdot \cos\frac{\pi}{n} = \pi R^2 \cdot \underbrace{\cos 0}_{=1} = \pi R^2$$

Ou seja, quando $n \to +\infty$, o polígono regular de n lados vai se aproximando de uma circunferência, cuja área será: $A = \pi R^2$.

07) $\gamma = 1380^\circ$.

$$\begin{array}{r|l} 1380^\circ & 360^\circ \\ \hline -1080^\circ & 3 \text{ VOLTAS} \\ \hline 300^\circ & \end{array}$$

$M = \text{sen } 1380^\circ \cdot \cos 1380^\circ$.

$= \text{sen } 300^\circ \cdot \cos 300^\circ$

$= -\text{sen } 60^\circ \cdot \cos 60^\circ = -\frac{\sqrt{3}}{2} \cdot \frac{1}{2} = -\frac{\sqrt{3}}{4}$.

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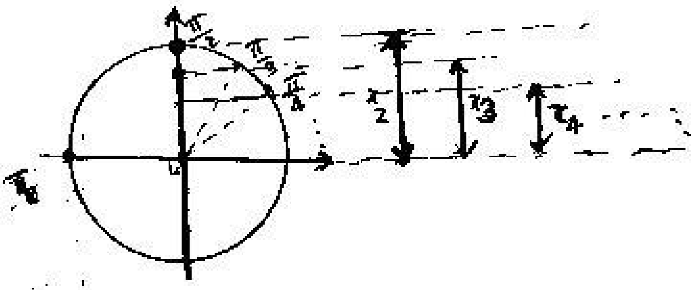
09) $(x_n)_{n \geq 2}$; $x_n = \text{sen } \frac{\pi}{n}$.

(a) como $|\text{sen } \alpha| \leq 1$, $\forall \alpha \in \mathbb{R}$, segue que

$\left|\text{sen } \frac{\pi}{n}\right| \leq 1$, $\forall n \in \mathbb{N}$, ou seja

a seq. (x_n) é limitada.

(b) 

Pelo esquema no ciclo notamos que (x_n) é decrescente.

(c) $\lim_{n \to \infty} x_n = \lim_{n \to \infty} \text{sen } \frac{\pi}{n} = 0$:

Dado $\varepsilon > 0$. Escolha $n_0 \in \mathbb{N}$ tal que $n_0 > \frac{\pi}{\varepsilon}$.

Assim, $\forall n \geq n_0$, segue que

$\frac{1}{n} \leq \frac{1}{n_0} < \frac{\varepsilon}{\pi}$, e daí;

$|x_n - 0| = \left|\text{sen } \frac{\pi}{n} - 0\right| = \left|\text{sen } \frac{\pi}{n}\right| \leq \left|\frac{\pi}{n}\right| = \frac{\pi}{n} < \cancel{\pi} \cdot \frac{\varepsilon}{\cancel{\pi}} = \varepsilon$

$\square$
