TRIGONOMETRIA. RESPOSTAS DA LISTA 03.
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01) Como $-1 \leq \operatorname{sen} x \leq 1$ e $-1 \leq \cos x \leq 1$, basta resolver as desigualdades:

(a) $-1 \leq 3k-2 \leq 1$. (b) $-1 \leq \dfrac{k+1}{k-1} \leq 1$.

etc.

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02) $\alpha \in 2^{\circ}q$.

$\csc \alpha = 5$. Achar os demais números trigonométricos.

- $\operatorname{sen}\alpha = \dfrac{1}{\csc\alpha} = \dfrac{1}{5}$
- $\operatorname{sen}^2\alpha + \cos^2\alpha = 1 \Rightarrow \cos\alpha = \pm\sqrt{1-\operatorname{sen}^2\alpha} = -\sqrt{1-\left(\frac{1}{5}\right)^2} = -\dfrac{2\sqrt{6}}{5}$.

$\alpha \in 2^{\circ}q$ e no $2^{\circ}q$ o cosseno é negativo.

$\Rightarrow \cos\alpha = -\dfrac{2\sqrt{6}}{5}$.

- $\sec\alpha = \dfrac{1}{\cos\alpha} = -\dfrac{5}{2\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}} = -\dfrac{5\sqrt{6}}{12}$.
- $\tan\alpha = \dfrac{\operatorname{sen}\alpha}{\cos\alpha} = \dfrac{\frac{1}{5}}{-\frac{2\sqrt{6}}{5}} = -\dfrac{1}{2\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}} = -\dfrac{\sqrt{6}}{12}$.
- $\cot\alpha = \dfrac{1}{\tan\alpha} = -\dfrac{12}{\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}} = -\dfrac{12\sqrt{6}}{6} = -2\sqrt{6}$.

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04) $\cos x = ?$ $m > 1$.

$\cot x = \dfrac{2\sqrt{m}}{m-1}$.

obs.: como $m > 1$, então $\dfrac{2\sqrt{m}}{m-1} > 0$, i.e., $\cot x > 0$. Logo, temos que $x \in 1^{\circ}q$ ou $x \in 3^{\circ}q$.

Como $1+\cot^2 x = \csc^2 x$, temos:

$$1+\left(\frac{2\sqrt{m}}{m-1}\right)^2 = \csc^2 x$$

$$\frac{(m-1)^2+4m}{(m-1)^2} = \csc^2 x \implies \csc^2 x = \frac{m^2-2m+1+4m}{(m-1)^2}$$

$$\implies \csc^2 x = \frac{m^2+2m+1}{(m-1)^2} = \left(\frac{m+1}{m-1}\right)^2.$$

Assim, $\operatorname{sen}^2 x = \frac{1}{\csc^2 x} = \left(\frac{m-1}{m+1}\right)^2. \implies \operatorname{sen} x = \pm\,\frac{m-1}{m+1}.$

Logo;

$$\operatorname{sen}^2 x + \cos^2 x = 1 \implies \cos^2 x = 1-\operatorname{sen}^2 x$$

$$\implies \cos^2 x = 1 - \left(\frac{m-1}{m+1}\right)^2 = \frac{(m+1)^2-(m-1)^2}{(m+1)^2}$$

$$= \frac{\cancel{m^2}+2m\cancel{+1}-\cancel{m^2}+2m\cancel{-1}}{(m+1)^2} = \frac{4m}{(m+1)^2}$$

$$\implies \boxed{\cos x = \pm\,\frac{2\sqrt{m}}{m+1}}. \qquad \square.$$

Por fim; apenas para conferir:

$$\cot x = \frac{\cos x}{\operatorname{sen} x} = \frac{\pm\,\frac{2\sqrt{m}}{\cancel{m+1}}}{\pm\,\frac{m-1}{\cancel{m+1}}} = \frac{2\sqrt{m}}{m-1}$$

08) $y = \dfrac{\sec\left(\frac{\pi}{2}-x\right)+\tan\left(\frac{\pi}{2}-x\right)}{\csc x} = \dfrac{\csc x + \cot x}{\csc x}$

$= \dfrac{\csc x}{\csc x} + \dfrac{\cot x}{\csc x} = 1 + \dfrac{\frac{\cos x}{\operatorname{sen} x}}{\frac{1}{\operatorname{sen} x}} =$

$= 1 + \dfrac{\cos x}{\cancel{\operatorname{sen} x}} \cdot \dfrac{\cancel{\operatorname{sen} x}}{1} = 1+\cos x \Rightarrow \boxed{y = 1+\cos x}.$

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09) $x \in 1^{o}q$; achar $\cot x$, sendo dado

$\cos x + \operatorname{sen} x \cdot \tan x = 3.$

$\Leftrightarrow \cos x + \operatorname{sen} x \cdot \dfrac{\operatorname{sen} x}{\cos x} = 3$

$\Leftrightarrow \dfrac{\cos^2 x + \operatorname{sen}^2 x}{\cos x} = 3$

$\Leftrightarrow \dfrac{1}{\cos x} = 3 \Leftrightarrow \sec x = 3.$

Como $1+\tan^2 x = \sec^2 x$, segue que

$\tan^2 x = \sec^2 x - 1 = (3)^2 - 1 = 8$

$\Rightarrow \tan x = \pm\sqrt{8} \Rightarrow \tan x = +2\sqrt{2}$ $(x \in 1^{o}q)$

Finalmente; $\cot x = \dfrac{1}{\tan x} = \dfrac{1}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{4}$ //

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10)

(a) $$\frac{\sec\alpha - \csc\alpha}{\sec\alpha + \csc\alpha} = \frac{\frac{1}{\cos\alpha} - \frac{1}{\operatorname{sen}\alpha}}{\frac{1}{\cos\alpha} + \frac{1}{\operatorname{sen}\alpha}} = \frac{\frac{\operatorname{sen}\alpha - \cos\alpha}{\cancel{\cos\alpha \cdot \operatorname{sen}\alpha}}}{\frac{\operatorname{sen}\alpha + \cos\alpha}{\cancel{\cos\alpha \cdot \operatorname{sen}\alpha}}} =$$

$$= \frac{\operatorname{sen}\alpha - \cos\alpha}{\operatorname{sen}\alpha + \cos\alpha} = \frac{\frac{\operatorname{sen}\alpha - \cos\alpha}{\cos\alpha}}{\frac{\operatorname{sen}\alpha + \cos\alpha}{\cos\alpha}} = \frac{\frac{\operatorname{sen}\alpha}{\cos\alpha} - \frac{\cancel{\cos\alpha}}{\cos\alpha}}{\frac{\operatorname{sen}\alpha}{\cos\alpha} + \frac{\cancel{\cos\alpha}}{\cos\alpha}} =$$

$$= \frac{\tan\alpha - 1}{\tan\alpha + 1}.$$

(b) $$\frac{\operatorname{sen}x}{1+\cos x} + \frac{1+\cos x}{\operatorname{sen}x} = \frac{\operatorname{sen}^2 x + (1+\cos x)^2}{\operatorname{sen}x\,(1+\cos x)} =$$

$$= \frac{\operatorname{sen}^2 x + 1 + 2\cos x + \cos^2 x}{\operatorname{sen}x\,(1+\cos x)} = \frac{2 + 2\cos x}{\operatorname{sen}x\,(1+\cos x)} =$$

$$= \frac{2\cdot\cancel{(1+\cos x)}}{\operatorname{sen}x\cdot\cancel{(1+\cos x)}} = \frac{2}{\operatorname{sen}x} = 2\cdot\csc x.$$

(d) $$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{\tan\theta + \sec\theta + \tan^2\theta - \sec^2\theta}{\tan\theta - \sec\theta + 1} =$$

$$= \frac{(\tan\theta + \sec\theta)\cancel{(1 + \tan\theta - \sec\theta)}}{\cancel{\tan\theta - \sec\theta + 1}} = \tan\theta + \sec\theta.$$

[obs.: neste item usamos que $1 + \tan^2\theta = \sec^2\theta$, e então $1 = \sec^2\theta - \tan^2\theta$.].