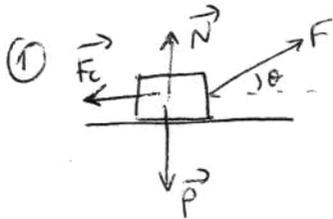


LISTA II.1



$\theta = 36,87^\circ$
 $m = 5 \text{ kg}$
 $\Delta x = 8 \text{ m}$
 $v = 1,2 \text{ m/s}$
 $\mu_c = 0,75$

- (a) W de todas as forças?
 (b) $W_f = ?$
 (c) W de força resultante?

(a) $W_N = W_P = 0$ pois as forças são \perp ao deslocamento

$W_{F_c} = -W_F$ pois a veloc. é constante \Rightarrow (b) $W_{\text{total}} = \Delta K = 0$

Como $F_R = 0$: $F_x - F_c = 0$ (c) $W_{\text{RES}} = F_{R_x} \Delta x = 0$ pois $F_R = 0$ (não tem \vec{a})

$F \cos \theta - \mu (P - F \sin \theta) = 0$

$F (\cos \theta + \mu \sin \theta) = \mu P$

$F = \frac{\mu P}{\cos \theta + \mu \sin \theta} = \frac{0,75 \cdot 50}{0,8 + 0,75 \cdot 0,6} = 30 \text{ N}$

Então $W_F = F \cos \theta \Delta x = 30 \cdot 0,8 \cdot 8 = 192 \text{ J}$ e $W_{F_c} = -192 \text{ J}$

(2) Se $a = 0,5 \text{ m/s}^2 \Rightarrow F_R \neq 0$: $F_x - F_c = ma$

$F \cos \theta - \mu (P - F \sin \theta) = ma$

$F (\cos \theta + \mu \sin \theta) = ma + \mu P$

$F = \frac{m(a + \mu g)}{\cos \theta + \mu \sin \theta} = \frac{5(0,5 + 0,75 \cdot 10)}{0,8 + 0,75 \cdot 0,6} = \frac{40}{1,25} = 32 \text{ N}$

Então $W_F = 32 \cdot 0,8 \cdot 8 = 205 \text{ J}$

$W_{F_c} = -\mu (P - F \sin \theta) \Delta x = -0,75 (50 - 32 \cdot 0,6) \cdot 8 = -185 \text{ J}$



$m = 27 \text{ kg}$
 $\Delta x = 91 \text{ m}$
 \vec{v} cte
 $\theta = -32^\circ$
 $\mu = 0,2$

$F \cos \theta - F_c = 0$ e $N - P - F \sin \theta = 0$

$F \cos \theta - \mu (P + F \sin \theta) = 0$

$F (\cos \theta - \mu \sin \theta) = \mu P$

$F = \frac{0,2 \cdot 270}{0,85 - 0,2 \cdot 0,53} = \frac{54}{0,74} = 73 \text{ N}$

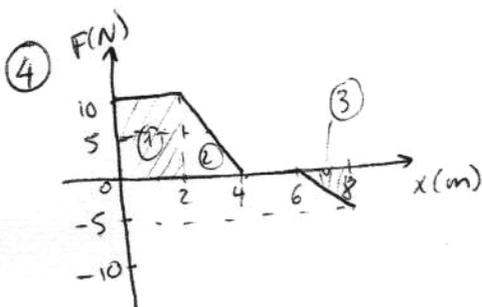
\Downarrow
 $W_F = F \cos 32^\circ \Delta x$
 $= 73 \cdot 0,85 \cdot 91 = 5644 \text{ J} //$

Calculamos o trabalho pela área do gráfico:

$W = (b_1 \times h_1) + \frac{(b_2 \times h_2)}{2} + 0 + \frac{(b_3 \times h_3)}{2}$
 ① ② ③

$W = (2 \times 10) + \frac{(2 \times 10)}{2} + \left[\frac{2 \times (-5)}{2} \right] = 20 + 10 - 5$

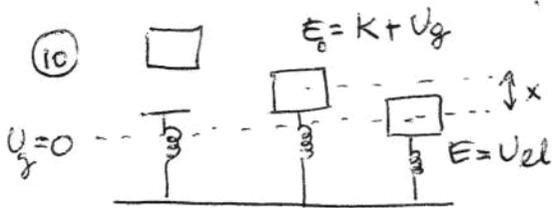
$W = 25 \text{ J}$



8) Problema 2: $v_0 = 0$
 $v = ?$

Do teorema trabalho-energia, podemos igualar o trabalho total realizado com a variação da energia cinética: $W = \Delta K = K - K_0 = \frac{1}{2} m (v^2 - v_0^2)$

$$W_T = W_F + W_{F_c} = 205 - 185 = 20 \text{ J} \quad 20 \cdot \frac{2}{5} = v^2 \quad v = \sqrt{8} = 2,8 \text{ m/s} //$$



$m = 250 \text{ g} = 0,25 \text{ kg}$
 $K = 25 \frac{\text{N}}{\text{cm}} = 250 \frac{\text{N}}{\text{m}}$
 $\lambda = 12 \text{ cm} = 0,12 \text{ m}$

- (a) $W_p = ?$ $W_p = mgx = 0,3 \text{ J}$
 (b) $W_{el} = ?$ $W_{el} = \frac{1}{2} Kx^2 (f) = -18 \text{ J}$
 (c) $v_{\text{max}} = ?$
 (d) se $v = 2 v_{\text{max}} \Rightarrow x = ?$

(c) Colocando o zero da energia potencial no ponto mais baixo do bloco (ver desenho), igualamos a energia mecânica: $E_0 = E$

$$\frac{1}{2} m v^2 + mgx = \frac{1}{2} K x^2 \quad *$$

$$v^2 + 2gx = \frac{K}{m} x^2$$

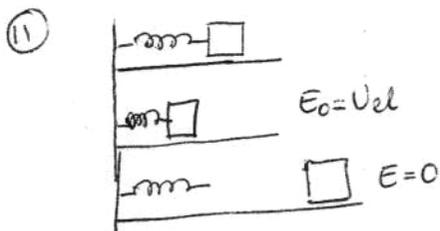
$$v = \sqrt{\frac{K}{m} x^2 - 2gx} = \sqrt{\frac{250}{0,25} \cdot 0,12^2 - 20 \cdot 0,12} = \sqrt{12} = 3,5 \text{ m/s}$$

Também podemos usar o teorema trabalho-energia: $W_T = \Delta K$

$$W_p + W_{el} = \frac{1}{2} m (v^2 - v_0^2) \quad 0,3 - 1,8 = -\frac{1}{2} \cdot 0,25 v_0^2 \quad v_0 = \sqrt{12} = 3,5 \text{ m/s}$$

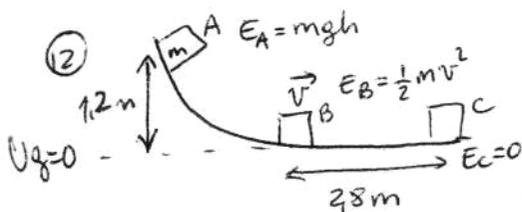
(d) Podemos achar a elongação quando $v = 2 v_{\text{max}}$ usando a expressão *:

$$\frac{1}{2} K x^2 - mgx - \frac{1}{2} m v^2 = 0 \quad 125 x^2 - 25 x - 6,1 = 0 \Rightarrow \dots$$



$m = 2 \text{ kg}$
 $x = 15 \text{ cm} = 0,15 \text{ m}$
 $\Delta x = 60 \text{ cm} = 0,6 \text{ m}$
 $K = 200 \frac{\text{N}}{\text{m}}$
 $\mu = ?$

$W_{\text{atrito}} = \Delta E$
 $\mu N \Delta x (-1) = 0 - U_{el}$
 $\mu mg \Delta x = \frac{1}{2} K x^2$
 $\mu = \frac{1}{2} \frac{K x^2}{mg \Delta x} = 0,19$



$P = 10 \text{ N}$
 $v_0 = 0$
 $v = 3,7 \text{ m/s}$
 $\Delta x = 2,8 \text{ m}$

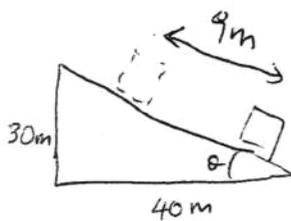
- (a) $\mu_c = ?$ $B \rightarrow C$
 (b) $W_{\text{atrito}} = ?$ $B \rightarrow C$
 (c) $W_{\text{atrito}} = ?$ $A \rightarrow B$

(a) $W_{\text{atrito}} = E_C - E_B$
 $-\mu mg \Delta x = 0 - \frac{1}{2} m v^2$
 $\mu = \frac{v^2}{2g \Delta x} = \frac{13,69}{56} = 0,24$

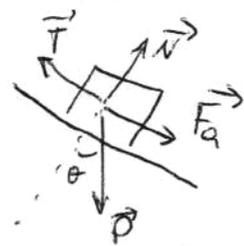
(b) $W_{\text{atrito}} = -\mu mg \Delta x$
 $= -0,24 \cdot 10 \cdot 0,6$
 $= -2,9 \text{ J}$

(c) $W_{\text{atrito}} = E_B - E_A$
 $= \frac{1}{2} m v^2 - mgh$
 $= 6,8 - 12$
 $= -5,2 \text{ J}$

15) $m = 1400 \text{ kg}$
 $v = 1,34 \text{ m/s}$
 $\mu = 0,4$



$\tan \theta = \frac{30}{40} \Rightarrow \theta = 36,9^\circ$



Como o bloco sobe com veloc. constante, a aceleração é nula e, portanto, a força resultante é nula. Por isso, o trabalho total realizado (que é igual ao trabalho da força resultante) é nulo:

$$W_T = W_p + W_N + W_{F_a} + W_T = 0$$

0 pois a normal é \perp ao deslocamento

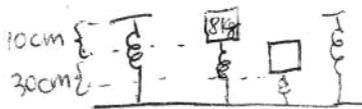
$$W_p = P \cdot \Delta x \cdot \cos(53^\circ + 180^\circ) = 14000 \cdot 9 \cdot (-0,6) = -75.600 \text{ J} = -75,6 \text{ KJ}$$

$$W_{F_a} = \mu \cdot N \cdot \Delta x \cdot \cos 180^\circ = \mu \cdot mg \cdot \cos 37^\circ \cdot \Delta x \cdot (-1) = 4.472,4 \text{ J} = -4,5 \text{ KJ}$$

$$W_T = -W_p - W_{F_a} = 130.472,4 \text{ J} = 130,5 \text{ KJ}$$

Mas $W_T = T \cdot \Delta x \Rightarrow T = \frac{130,5 \cdot 10^3}{9} = 14,5 \text{ KN} \Rightarrow P = F \cdot v = 19,4 \text{ KW}$

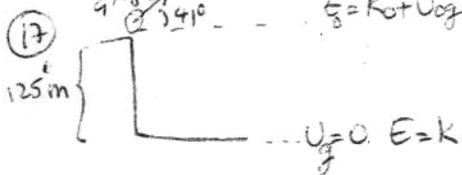
16)



(a) Bloco em equilíbrio $\Rightarrow F_{el} - P = 0 \Rightarrow kx = mg \Rightarrow k = \frac{mg}{x} = \frac{800 \text{ N}}{x}$

(b) $U_{el} = \frac{kx^2}{2} = \frac{800 \cdot (0,4)^2}{2} = 64 \text{ J}$

(c) $U_g = U_{el} \Rightarrow mgh = 64 \Rightarrow h = \frac{64}{80} = 0,8 \text{ m} = 80 \text{ cm}$



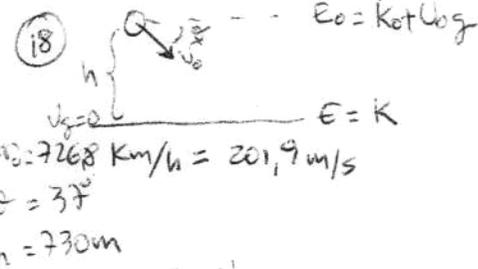
(a) $K_0 = \frac{1}{2} m v_0^2 = 27000 \text{ J} = 27 \text{ KJ}$

(b) $U_{0g} = mgh = 3000 \text{ J} = 3 \text{ KJ}$

(c) Ao atingir o solo: $E = E_0 \Rightarrow K = 30 \text{ KJ}$

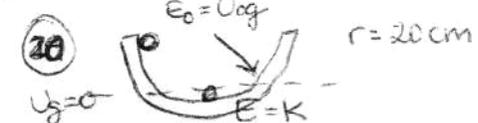
(d) Não: $\frac{1}{2} m v^2 = 30000 \Rightarrow v = 158 \text{ m/s}$

$E = E_0$
 $\frac{1}{2} m v^2 = \frac{1}{2} m v_0^2 + mgh \Rightarrow v = \sqrt{2gh + v_0^2}$

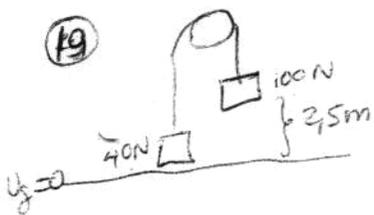


Como $E = E_0 \Rightarrow v = \sqrt{2gh + v_0^2} = 235 \text{ m/s}$

A veloc. inicial está ao quadrado \Rightarrow não importa o sentido do lançamento!



$E = E_0$
 $\frac{1}{2} m v^2 = mgh \Rightarrow v = \sqrt{2gr} = 2 \text{ m/s}$



$$E_0 = m_2gh$$

$$E = m_1gh + \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2$$

$$E = E_0$$

$$\frac{1}{2}v^2(m_1+m_2) = gh(m_2-m_1)$$

$$v^2 = \frac{2gh(m_2-m_1)}{m_1+m_2}$$

$$v = \sqrt{\frac{50 \cdot 6}{14}} = 4.6 \text{ m/s}$$

(21) $F = 270 \text{ N} \Rightarrow \Delta x = 2 \text{ cm} \Rightarrow F = k\Delta x \quad k = \frac{270}{0.02} = 13.500 \frac{\text{N}}{\text{m}}$

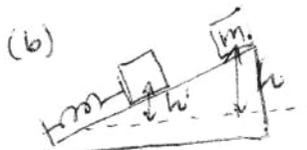


(a) $\sin\theta = \frac{h}{d} \quad h = d \sin\theta$

$$E = U_{el} \quad E = E_0 \quad \frac{1}{2}k\Delta x^2 = mgd \sin\theta$$

$$d = \frac{k\Delta x^2}{2mg \sin\theta} = \frac{13500 \cdot (0.055)^2}{2 \cdot 12 \cdot 10 \cdot 0.95} = 0.34 \text{ m}$$

$$d = 34 \text{ cm}$$



$$E' = \frac{1}{2}mv'^2 + mgh'$$

$$E' = E_0$$

$$\frac{1}{2}mv'^2 + mgd' \sin\theta = mgd \sin\theta$$

$$v'^2 = 2g \sin\theta (d - d')$$

$$v' = \sqrt{2 \cdot 10 \cdot 0.95 (0.34 - 0.055)} = 1.69 \text{ m/s}$$



$$E'' = \frac{1}{2}mv''^2 + mgh'' + \frac{1}{2}k\Delta x''^2$$

$$E'' = E_0$$

$$\frac{1}{2}mv''^2 + mgd'' \sin\theta + \frac{1}{2}k\Delta x''^2 = mgd \sin\theta$$

$$3 \text{ cm } mv''^2 = 2mg \sin\theta (d - d'') - k\Delta x''^2$$

$$v'' = \sqrt{\frac{2 \cdot 10 \cdot 0.95 (0.34 - 0.025) - 13500 \cdot 0.03^2}{m}}$$

$$v'' = 1.46 \text{ m/s}$$

(22) Se houver atrito, a diferença entre as energias mecânicas será o trabalho da força de atrito. ($\mu = 0.2$)

(a) $W_{at} = E - E_0 \Rightarrow \mu mg \cos\theta d (-1) = \frac{1}{2}k\Delta x^2 - mgd \sin\theta$

$$\frac{1}{2}k\Delta x^2 = mgd (\sin\theta - \mu \cos\theta)$$

$$d = \frac{k\Delta x^2}{2mg(\sin\theta - \mu \cos\theta)} = 0.52 \text{ m}$$

$$d = 52 \text{ cm}$$

(b) $W_{at} = E' - E_0 \Rightarrow \mu mg \cos\theta (d - d') (-1) = \frac{1}{2}mv'^2 + mgh' - mgh$

$$\frac{1}{2}mv'^2 = -\mu mg \cos\theta (d - d') - mgh' + mgh$$

$$v'^2 = 2g [d \sin\theta - d' \sin\theta - \mu \cos\theta (d - d')] \quad d' = 5.5 \text{ cm} = 0.055$$

$$d = 52 \text{ cm} = 0.52$$

$$v' = 1.74 \text{ m/s}$$

(c) $W_{at} = E'' - E_0 \Rightarrow \mu mg \cos\theta (d - d'') (-1) = \frac{1}{2}mv''^2 + mgh'' + \frac{1}{2}k\Delta x''^2 - mgh$

$$\frac{1}{2}mv''^2 = mgh - mgh'' - \mu mg \cos\theta (d - d'') - \frac{1}{2}k\Delta x''^2$$

$$v''^2 = 2g [d \sin\theta - d'' \sin\theta - \mu \cos\theta (d - d'')] - \frac{k}{m} \Delta x''^2$$

$$v''^2 = 2g [\sin\theta (d - d'') - \mu \cos\theta (d - d'')] - \frac{k}{m} \Delta x''^2$$

$$v''^2 = 2g (d - d'') (\sin\theta - \mu \cos\theta) - \frac{k}{m} \Delta x''^2$$

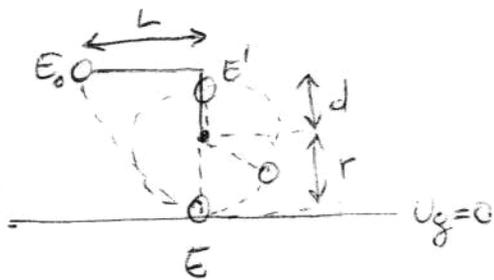
$$v'' = 1.49 \text{ m/s}$$

$$d'' = 2.5 \text{ cm} = 0.025 \text{ m}$$

$$d = 52 \text{ cm} = 0.52 \text{ m}$$

$$\Delta x'' = 3 \text{ cm} = 0.03 \text{ m}$$

23) $L = 1,2 \text{ m}$
 $d = 0,75 \text{ m}$
 $r = L - d = 0,45 \text{ m}$



$$E_0 = mgL$$

$$E = \frac{1}{2}mv^2$$

(a) $E = E_0$
 $\frac{1}{2}mv^2 = mgL$
 $v = \sqrt{2gL}$
 $v = 4,9 \text{ m/s}$

(b) $E' = E_0$
 $mg(2r) + \frac{1}{2}mv^2 = mgL$
 $v^2 = 2g(L - 2r)$
 $v' = 2,45 \text{ m/s}$

Mostrar que
 (c) $d > 3L/5$

Para dar uma volta, a força centrípeta deve ser, pelo menos, igual ao peso (tração nula):

$$F_c > mg$$

$$\frac{mv^2}{r} > mg \quad v^2 > gr$$

$$v^2 > gr \quad v > \sqrt{gr}$$

$$2g(L - 2r) > gr$$

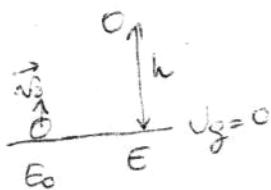
$$2L - 4r > r$$

$$2L > 5r$$

$$2L > 5(L - d)$$

$$5d > 3L \quad d > \frac{3L}{5}$$

24) $m = 60g = 0,06 \text{ kg}$
 $v_0 = 120 \text{ m/s}$
 $h = 680 \text{ cm}$



$$E_0 = K_0 = \frac{1}{2}mv_0^2$$

$$E = U = mgh$$

(a) $W_{\text{atr}} = E - E_0 = mgh - \frac{1}{2}mv_0^2 = 408 - 432 = -24 \text{ J}$

(b) $W_{\text{atr}} = E' - E = \frac{1}{2}mv^2 - mgh$

$$2\left(\frac{W_{\text{atr}}}{m} + gh\right) = v^2 \quad v' = \sqrt{2\left(\frac{-24}{0,06} + 6800\right)} = 113,14 \text{ m/s}$$

25) $m = 4 \text{ kg}$

$$F = 6,4 \text{ N} \Rightarrow \Delta x = 2 \text{ cm} \Rightarrow F = k\Delta x \quad k = \frac{6,4}{0,02} = 320 \frac{\text{N}}{\text{m}}$$

$$\Delta x' = 8 \text{ cm} = 0,08 \text{ m}$$

(a) $E = E_0 \quad \frac{1}{2}mv^2 = \frac{1}{2}k\Delta x'^2 \quad v^2 = \frac{k}{m}\Delta x'^2 \quad v = 0,7 \text{ m/s}$

(b) Se atrito ($\mu_e = 0,6 \quad \mu_c = 0,4$): $W_{\text{atr}} = E - E_0 = \frac{1}{2}k\Delta x''^2 - \frac{1}{2}mv^2$

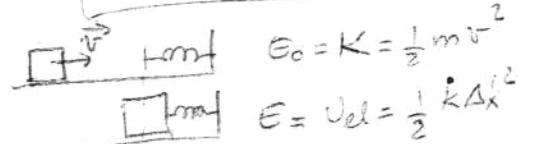
$$\mu mg\Delta x''(1-\mu) = \frac{1}{2}(k\Delta x''^2 - mv^2)$$

$$k\Delta x''^2 + 2\mu mg\Delta x'' - mv^2 = 0$$

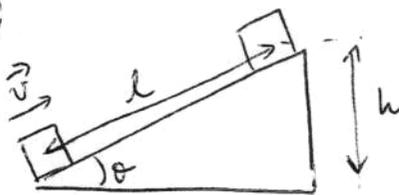
$$\Delta x'' = \frac{-352 \pm \sqrt{352^2 - 4 \cdot 320 \cdot (-1,96)}}{2 \cdot 320} = \frac{-352 \pm 355,5}{640}$$

$$\Delta x'' = 5,5 \cdot 10^{-3} \text{ m} = 0,5 \text{ cm}$$

(c) $W_{\text{atr}} = \frac{1}{2}320 \cdot (0,005)^2 - \frac{1}{2}4 \cdot (0,7)^2$
 $= 4 \cdot 10^{-3} - 0,98 = -0,976 \text{ J}$



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$m = 4 \text{ kg}$
 $\theta = 30^\circ$
 $K = 128 \text{ J}$
 $\mu = 0,3$
 $l = ?$

$$128 = \frac{1}{2} 4 v^2 \quad v = \sqrt{\frac{128}{2}} = 8 \text{ m/s}$$

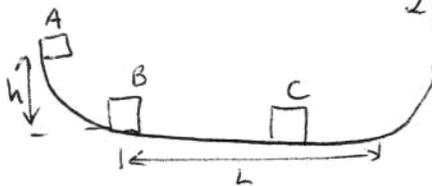
$$W_{\text{atr}} = E - E_0 \quad -\mu N l = mgh - \frac{1}{2} m v^2 \quad -\mu mg \cos \theta l = mgh - \frac{1}{2} m v^2$$

Mas $l \sin \theta = h \Rightarrow -\mu g l \cos \theta = g l \sin \theta - \frac{v^2}{2}$

$$\frac{v^2}{2} = g l \sin \theta (1 + \mu)$$

$$l = \frac{v^2}{2 g \sin \theta (1 + \mu)} = \frac{64}{10 \cdot 1,3} = 4,9 \text{ m} \quad (h = 2,5 \text{ m})$$

27 $\mu = 0,23$
 $h = 1 \text{ m}$
 $L = 2 \text{ m}$



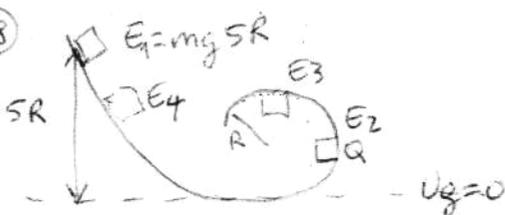
A \rightarrow B: $E_A = E_B \quad mgh = \frac{1}{2} m v^2 \quad v = \sqrt{2gh} = 4,5 \text{ m/s}$

B \rightarrow C: $W_{\text{atr}} = E_C - E_B = 0 - \frac{1}{2} m v^2$

$$-\mu mg l = -\frac{1}{2} m v^2 \quad l = \frac{v^2}{2 \mu g} = \frac{20}{4,6} = 4,3 \text{ m} \Rightarrow \text{vai, volta, e pára em } 0,3 \text{ m}$$

Se $\mu' = \frac{\mu}{g} \quad l' = 9l = 38,7 \Rightarrow \text{vai e volta } 9 \text{ vezes e pára em } 0,7 \text{ m}$

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$$E_1 = U_{g1} = mgSR$$

$$E_2 = U_{g2} + \frac{1}{2} m v^2 = mgR + \frac{1}{2} m v^2$$

Acharemos v igualando as em mecânicas
 $E_1 = E_2$

$$mgSR = mgR + \frac{1}{2} m v^2 \quad 4gR = \frac{v^2}{2} \quad v = \sqrt{8gR}$$

(a) $v_B = 0$

F em Q: $F_c = \frac{mv^2}{R} = \frac{m8gR}{R} = 8mg$

(b) Perder contato $\Rightarrow F_c = mg \Rightarrow \frac{mv^2}{R} = mg \quad v^2 = gR \Rightarrow E_3 = K + U_{g3}$

$$E_4 = E_3 \quad mgh = \frac{1}{2} m v^2 + mg2R \quad h = \frac{5}{2} R = 2,5R$$