

LISTA 1 – Integrais Definidas, Indefinidas e Impróprias

1. Calcule os seguintes integrais

(a) $\int (3x^4 - 5x^2 + x) dx$

(b) $\int \frac{x+1}{\sqrt{x}} dx$

(c) $\int \frac{\text{sen } x}{\cos^2 x} dx$

(d) $\int \frac{2}{\sqrt{x}} dx$

(e) $\int (t^2 + 1)^2 dt$

(f) $\int \frac{x^3 + 3}{x^2} dx$

(g) $\int \sqrt[3]{x}(x-4) dx$

(h) $\int 8x^3 + \frac{1}{2x^2} dx$

(i) $\int (x-4)(x+4) dx$

(j) $\int \frac{x^2 - 1}{x^{3/2}} dx$

(k) $\int (x+1)(3x-2) dx$

(l) $\int y^2 \sqrt{y} dy$

(m) $\int \frac{\cos x}{1 - \cos^2 x} dx$

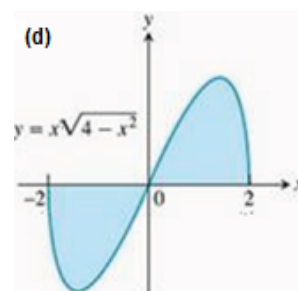
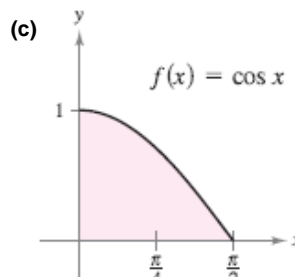
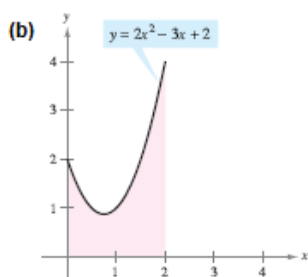
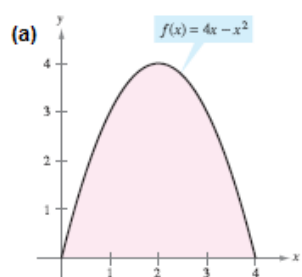
(n) $\int (\tan^2 y + 1) dy$

(o) $\int \frac{1}{x^2} + \frac{1}{x\sqrt{x}} + 2 dx$

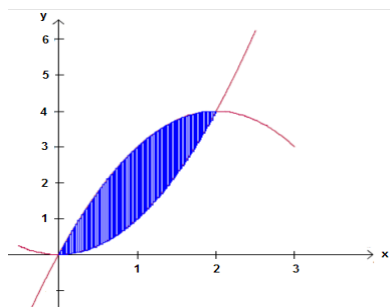
(p) $\int (5x + \cos x) dx$

(q) $\int e^x - \sec^2 x dx$

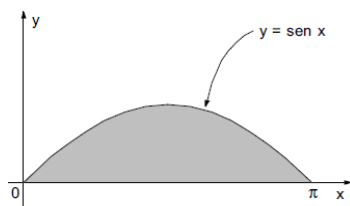
2. Calcule as áreas delimitadas abaixo por $f(x)$ aplicando o conceito de integração definida



(e) Calcule a área delimitada pelas curvas $f(x) = -x^2 + 4x$ e $g(x) = x^2$. As interseções ocorrem em $x = 0$ e $x = 2$.

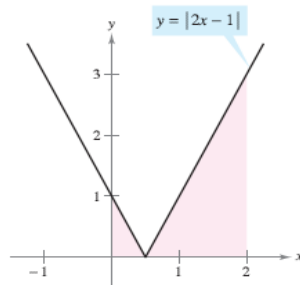


(f) Calcular a área compreendida entre a curva $y = \text{sen } x$ e o eixo x , para $0 \leq x \leq \pi$.



(g) Calcule a área hachurada abaixo de acordo com a seguinte definição para a função $f(x)$:

$$|2x - 1| = \begin{cases} -(2x - 1), & x < \frac{1}{2} \\ 2x - 1, & x \geq \frac{1}{2} \end{cases}$$



3. Calcule as integrais definidas

- (a) $\int_1^3 (-x^2 + 4x - 3) dx$ (b) $\int_0^1 t^{1/3} - t^{2/3} dt$ (c) $\int_{-\pi}^{\pi} \text{sen } x dx$ (d) $\int_{-1}^1 |x| dx$
 (e) $\int_1^4 3\sqrt{x} dx$ (f) $\int_0^{\pi} \sec^2 x dx$ (g) $\int_1^2 \left(\frac{3}{x^2} - 1 \right) dx$

4. Calcule as integrais abaixo utilizando o método da integração por substituição

- (a) $\int (x^2 + 1)^2 (2x) dx$ (b) $\int 5 \cos 5x dx$ (c) $\int \sqrt{2x-1} dx$ (d) $\int x\sqrt{2x-1} dx$
 (e) $\int \text{sen}^2 3x \cos 3x dx$ (f) $\int (2x+1)(x^2+x) dx$ (g) $\int 3x^2 \sqrt{x^3-2} dx$ (h) $\int \frac{-4x}{(1-2x^2)^2} dx$
 (i) $\int_1^5 \frac{x}{\sqrt{2x-1}} dx$ (j) $\int \left(1 + \frac{1}{t}\right)^3 \left(\frac{1}{t^2}\right) dt$ (k) $\int \tan^4 x \sec^2 x dx$ (l) $\int \cos(x+5) dx$
 (m) $\int \frac{\ln x dx}{x}$ (n) $\int \frac{dx}{x \ln x}$ (o) $\int \frac{5x^4 + 1}{x^5 + x} dx$ (p) $\int \frac{e^x}{1+e^x} dx$
 (q) $\int e^x \cos 2e^x dx$ (r) $\int \frac{3dx}{x \ln^2 3x}$

5. Calcule as integrais abaixo utilizando o método da integração por partes

- (a) $\int x \ln x dx$ (b) $\int x^3 e^{x^2} dx$ (c) $\int x \cos x dx$ (d) $\int (x^2 + 3x) \text{sen } x dx$
 (e) $\int 2x \ln x dx$ (f) $\int x \text{sen } 5x dx$ (g) $\int e^x \cos x dx$ (h) $\int \cos^3 x dx$
 (i) $\int (x^2 - 5x) e^x dx$ (j) $\int e^{2x} \cos 3x dx$ (k) $\int \sec^3 x dx$

6. Calcule as integrais racionais pelo método de frações parciais

- (a) $\int \frac{x-1}{x^3-x^2-2x} dx$ (b) $\int \frac{x^3-1}{x^2(x-2)^3} dx$ (c) $\int \frac{dx}{x^3+3x^2}$ (d) $\int \frac{4w-11}{2w^2+7w-4} dw$
 (e) $\int \frac{5x-2}{x^2-4} dx$ (f) $\int \frac{5x^2+20x+6}{x^3+2x^2+x} dx$ (g) $\int \frac{8x^3+13x}{(x^2+2)^2} dx$ (h) $\int \frac{2x^3-4x-8}{(x^2-x)(x^2+4)} dx$

7. Calcule as integrais impróprias

$$(a) \int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

$$(b) \int_1^{\infty} \frac{x+1}{x^2+2x} dx$$

$$(c) \int_4^{\infty} e^{-y/2} dy$$

$$(d) \int_{-\infty}^{\infty} x e^{-x^2} dx$$

$$(e) \int_1^9 \frac{dx}{\sqrt[3]{x-9}}$$

$$(f) \int_{-\infty}^{\infty} \frac{2dx}{x^2+9}$$

$$(g) \int_1^4 \frac{dx}{(x-2)^2}$$

RESPOSTAS

$$1. (a) \frac{3}{5}x^5 - \frac{5}{3}x^3 + \frac{1}{2}x^2 + C \quad (b) \frac{2}{3}\sqrt{x}(x+3) + C \quad (c) \sec x + C \quad (d) 4\sqrt{x} + C \quad (e) \frac{1}{5}t^5 + \frac{2}{3}t^3 + t + C$$

$$(f) \frac{1}{2}x^2 - \frac{3}{x} + C \quad (g) \frac{3}{7}x^{7/3} - 3x^{4/3} + C \quad (h) 2x^4 - \frac{1}{2x} + C \quad (i) \frac{1}{3}x^3 - 16x + C \quad (j) \frac{2(x^2+3)}{3\sqrt{x}} + C$$

$$(k) x^3 + \frac{1}{2}x^2 - 2x + C \quad (l) \frac{2}{7}y^{7/2} + C \quad (m) -\csc x + C \quad (n) \tan y + C \quad (o) -\frac{1}{x} - \frac{2}{\sqrt{x}} + 2x + C$$

$$(p) \frac{5x^2}{2} + \operatorname{sen} x + C \quad (q) e^x - \operatorname{tg} x + C$$

$$2. (a) 32/3 \text{ u.a.} \quad (b) 10/3 \text{ u.a.} \quad (c) 1 \text{ u.a.} \quad (d) 16/3 \text{ u.a.} \quad (e) 8/3 \text{ u.a.} \quad (f) 2 \text{ u.a.} \quad (g) 5/2 \text{ u.a.}$$

$$3. (a) 4/3 \quad (b) -27/20 \quad (c) 0 \quad (d) 1 \quad (e) 14 \quad (f) 1 \quad (g) 1/2$$

$$4. (a) \frac{1}{3}(x^2+1)^3 + C \quad (b) \operatorname{sen} 5x + C \quad (c) \frac{1}{3}(2x-1)^{3/2} + C \quad (d) \frac{1}{10}(2x-1)^{5/2} + \frac{1}{6}(2x-1)^{3/2} + C$$

$$(e) \frac{1}{9}\operatorname{sen}^3 3x + C \quad (f) \frac{(x^2+x)^2}{2} + C \quad (g) \frac{2}{3}(x^3-2)^{2/3} + C \quad (h) -\frac{1}{1-2x^2} + C \quad (i) 16/3$$

$$(j) -\frac{1}{4}\left(1+\frac{1}{t}\right)^4 + C \quad (k) \frac{1}{5}\tan^5 5x + C \quad (l) \operatorname{sen}(x+5) + C \quad (m) \frac{\ln^2 x}{2} + C \quad (n) \ln(\ln x) + C$$

$$(o) \ln(x^5+x) + C \quad (p) \ln(1+e^x) + C \quad (q) \frac{1}{2}\operatorname{sen} 2e^x + C \quad (r) -\frac{3}{\ln 3x} + C$$

$$5. (a) \frac{x^2}{2}\ln x - \frac{x^2}{4} + C \quad (b) \frac{1}{2}x^2 e^{x^2} - \frac{1}{2}e^{x^2} + C \quad (c) x \operatorname{sen} x + \cos x + C$$

$$(d) -(x^2+3x-2)\cos x + (2x+3)\operatorname{sen} x + C \quad (e) x^2 \ln x - \frac{x^2}{2} + C \quad (f) -\frac{x}{5}\cos 5x + \frac{1}{25}\operatorname{sen} 5x + C$$

$$(g) \frac{e^x \operatorname{sen} x + e^x \cos x}{2} + C \quad (h) \frac{1}{3}\cos^2 x \operatorname{sen} x + \frac{2}{3}\operatorname{sen} x + C \quad (i) (x^2-7x+7)e^x + C$$

$$(j) \frac{e^{2x}}{13}(3\operatorname{sen} 3x + 2\cos 3x) + C \quad (k) \frac{1}{2}\sec x \operatorname{tg} x + \frac{1}{2}\ln|\sec x + \operatorname{tg} x| + C$$

6. (a) $\frac{1}{6} \ln \left| \frac{Cx^3(x-2)}{(x+1)^4} \right|$ (b) $\frac{-11x^2 + 17x - 4}{8x(x-2)^2} + \frac{3}{16} \ln \left| \frac{x}{x-2} \right| + C$ (c) $\frac{1}{9} \ln \left| \frac{x+3}{x} \right| - \frac{1}{3x} + C$

(d) $\ln \left| \frac{C(w+4)^3}{2w-1} \right|$ (e) $\ln |C(x-2)^2(x+2)^3|$ (f) $\ln \left| \frac{x^6}{x+1} \right| - \frac{9}{x+1} + C$ (g) $4 \ln(x^2+2) + \frac{3}{2(x^2+2)} + C$

(h) $2 \ln|x| - 2 \ln|x-1| + \ln(x^2+4) + 2 \operatorname{arctg} \frac{x}{2} + C$

7. (a) $\pi/2$ (b) ∞ diverge (c) $2e^{-2}$ (d) 0 (e) -6 (f) $2\pi/3$ (g) ∞ diverge