UNIVERSIDADE FEDERAL DE PELOTAS
CENTRO DAS ENGENHARIAS

Disciplina: *Vetores e Álgebra linear* **Prof.:** *Germán Suazo*

Lista 05

1. Calcule os determinantes a seguir, utilizando a expansão por co-fatores pela linha ou coluna que lhe parecer adequada:

a. $\begin{vmatrix} 5 & 2 & 2 \\ -1 & 1 & 2 \\ 3 & 0 & 0 \end{vmatrix}$; b. $\begin{vmatrix} 1 & 1 & -1 \\ 2 & 0 & 1 \\ 3 & -2 & 1 \end{vmatrix}$; c. $\begin{vmatrix} \cos(\theta) & \operatorname{sen}(\theta) & \tan(\theta) \\ 0 & \cos(\theta) & -\operatorname{sen}(\theta) \\ 0 & \operatorname{sen}(\theta) & \cos(\theta) \end{vmatrix}$;

d. $\begin{vmatrix} a & b & 0 \\ 0 & a & b \\ a & 0 & b \end{vmatrix}$; e. $\begin{vmatrix} 0 & a & 0 \\ b & c & d \\ 0 & e & 0 \end{vmatrix}$; f. $\begin{vmatrix} 1 & -1 & 0 & 3 \\ 2 & 5 & 2 & 6 \\ 0 & -1 & 1 & 4 \\ 1 & 4 & 2 & 1 \end{vmatrix}$;

g. $\begin{vmatrix} 2 & 0 & 3 & -1 \\ 1 & 0 & 2 & 2 \\ 0 & -1 & 1 & 4 \\ 2 & 0 & 1 & -3 \end{vmatrix}$; h. $\begin{vmatrix} 0 & 0 & 0 & a \\ 0 & 0 & b & c \\ 0 & d & e & f \\ g & h & i & j \end{vmatrix}$; i. $\begin{vmatrix} 1 & -1 & 2 & 3 \\ -1 & 2 & 1 & 2 \\ 3 & 0 & 7 & 6 \\ 2 & 2 & 8 & 3 \end{vmatrix}$.

2. Calcule os determinantes a seguir de maneira direta, aplicando as propriedades dos determinantes:

a. $\begin{vmatrix} 1 & 1 & 1 \\ 3 & 0 & -2 \\ 2 & 2 & 2 \end{vmatrix}$; b. $\begin{vmatrix} 3 & 1 & 0 \\ 0 & -2 & 5 \\ 0 & 0 & 4 \end{vmatrix}$; c. $\begin{vmatrix} 0 & 0 & 1 \\ 0 & 5 & 2 \\ 3 & -1 & 4 \end{vmatrix}$;

d. $\begin{vmatrix} 2 & 3 & -4 \\ 1 & -3 & -2 \\ -1 & 5 & 2 \end{vmatrix}$; e. $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 4 & 1 \\ 1 & 6 & 4 \end{vmatrix}$; f. $\begin{vmatrix} 4 & 1 & 3 \\ -2 & 0 & -2 \\ 5 & 4 & 1 \end{vmatrix}$;

g. $\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$; h. $\begin{vmatrix} 0 & 2 & 0 & 0 \\ -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 \end{vmatrix}$; i. $\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{vmatrix}$

RESPOSTAS:

1. a. terceira linha: $3\cdot\begin{vmatrix}2 & 2\\ 1 & 2\end{vmatrix}-0\cdot\begin{vmatrix}5 & 2\\ -1 & 2\end{vmatrix}+0\cdot\begin{vmatrix}5 & 2\\ -1 & 1\end{vmatrix}=3\cdot(2)=6$;

1. b. segunda linha: $-2\cdot\begin{vmatrix}1 & -1\\ -2 & 1\end{vmatrix}+0\cdot\begin{vmatrix}5 & 2\\ -1 & 2\end{vmatrix}-1\cdot\begin{vmatrix}1 & 1\\ 3 & -2\end{vmatrix}=-2\cdot(-1)-1\cdot(-5)=7$;

1. c. primeira coluna:

$$\cos(\theta)\cdot\begin{vmatrix}\cos(\theta) & -\operatorname{sen}(\theta)\\ \operatorname{sen}(\theta) & \cos(\theta)\end{vmatrix}-0+0=\cos(\theta)\cdot\left(\cos^2(\theta)+\operatorname{sen}^2(\theta)\right)=\cos(\theta);$$

1. d. primeira linha: $a\cdot\begin{vmatrix}a & b\\ 0 & b\end{vmatrix}-b\cdot\begin{vmatrix}0 & b\\ a & b\end{vmatrix}+0\cdot\begin{vmatrix}0 & b\\ a & b\end{vmatrix}=a^2b+ab^2=ab(a+b)$;

1. e. primeira coluna: $0\cdot\begin{vmatrix}c & d\\ e & 0\end{vmatrix}-b\cdot\begin{vmatrix}a & 0\\ e & 0\end{vmatrix}+0\cdot\begin{vmatrix}a & 0\\ c & d\end{vmatrix}=0$;

1. f. primeira linha $1\cdot\begin{vmatrix}5 & 2 & 6\\ -1 & 1 & 4\\ 4 & 2 & 1\end{vmatrix}-(-1)\cdot\begin{vmatrix}2 & 2 & 6\\ 0 & 1 & 4\\ 1 & 2 & 1\end{vmatrix}+0\cdot\begin{vmatrix}2 & 5 & 6\\ 0 & -1 & 4\\ 1 & 4 & 1\end{vmatrix}-3\cdot\begin{vmatrix}2 & 5 & 2\\ 0 & -1 & 1\\ 1 & 4 & 2\end{vmatrix}$;

Agora,

$$\begin{vmatrix}5 & 2 & 6\\ -1 & 1 & 4\\ 4 & 2 & 1\end{vmatrix}=5\cdot\begin{vmatrix}1 & 4\\ 2 & 1\end{vmatrix}-2\cdot\begin{vmatrix}-1 & 4\\ 4 & 1\end{vmatrix}+6\cdot\begin{vmatrix}-1 & 1\\ 4 & 2\end{vmatrix}=5\cdot(-7)-2\cdot(-17)+6\cdot(-6)=-37;$$

$$\begin{vmatrix}2 & 2 & 6\\ 0 & 1 & 4\\ 1 & 2 & 1\end{vmatrix}=2\cdot\begin{vmatrix}1 & 4\\ 2 & 1\end{vmatrix}-0\cdot\begin{vmatrix}2 & 6\\ 2 & 1\end{vmatrix}+1\cdot\begin{vmatrix}2 & 6\\ 1 & 4\end{vmatrix}=2\cdot(-7)-0+1\cdot(2)=-12;$$

$$\begin{vmatrix}2 & 5 & 2\\ 0 & -1 & 1\\ 1 & 4 & 2\end{vmatrix}=2\cdot\begin{vmatrix}-1 & 1\\ 4 & 2\end{vmatrix}-0\cdot\begin{vmatrix}5 & 2\\ 4 & 2\end{vmatrix}+1\cdot\begin{vmatrix}5 & 2\\ -1 & 1\end{vmatrix}=2\cdot(-6)-0+1\cdot(7)=-5.$$

O determinante original fica $1\cdot(-37)-(-1)\cdot(-12)+0-3\cdot(-5)=-34$.

1. g. segunda coluna $-0+0-(-1)\cdot\begin{vmatrix}2 & 3 & -1\\ 1 & 2 & 2\\ 2 & 1 & -3\end{vmatrix}-0$;

Agora,

$$\begin{vmatrix}2 & 3 & -1\\ 1 & 2 & 2\\ 2 & 1 & -3\end{vmatrix}=2\cdot\begin{vmatrix}2 & 2\\ 1 & -3\end{vmatrix}-3\cdot\begin{vmatrix}1 & 2\\ 2 & -3\end{vmatrix}+(-1)\cdot\begin{vmatrix}1 & 2\\ 2 & 1\end{vmatrix}=2\cdot(-8)-3\cdot(-7)+(-1)\cdot(-3)=8;$$

Agora, o determinante fica $-(-1)\cdot\begin{vmatrix}2 & 3 & -1\\ 1 & 2 & 2\\ 2 & 1 & -3\end{vmatrix}=8$.

1. h. primeira linha: $(-a)\cdot\begin{vmatrix}0 & 0 & b\\ 0 & d & e\\ g & h & i\end{vmatrix}=(-a)\cdot b\cdot\begin{vmatrix}0 & d\\ g & h\end{vmatrix}=abdg$.

1. i. terceira linha: $3\cdot\begin{vmatrix}-1 & 2 & 3\\ 2 & 1 & 2\\ 2 & 8 & 3\end{vmatrix}-0+7\cdot\begin{vmatrix}1 & -1 & 3\\ -1 & 2 & 2\\ 2 & 2 & 3\end{vmatrix}-6\cdot\begin{vmatrix}1 & -1 & 2\\ -1 & 2 & 1\\ 2 & 2 & 8\end{vmatrix}$;

Agora,

$$\begin{vmatrix}-1 & 2 & 3\\ 2 & 1 & 2\\ 2 & 8 & 3\end{vmatrix}=(-1)\cdot\begin{vmatrix}1 & 2\\ 8 & 3\end{vmatrix}-2\cdot\begin{vmatrix}2 & 2\\ 2 & 3\end{vmatrix}+3\cdot\begin{vmatrix}2 & 1\\ 2 & 8\end{vmatrix}=(-1)\cdot(-13)-2\cdot(2)+3\cdot(14)=51;$$

$$\begin{vmatrix}1 & -1 & 3\\ -1 & 2 & 2\\ 2 & 2 & 3\end{vmatrix}=1\cdot\begin{vmatrix}2 & 2\\ 2 & 3\end{vmatrix}-(-1)\cdot\begin{vmatrix}-1 & 2\\ 2 & 3\end{vmatrix}+3\cdot\begin{vmatrix}-1 & 2\\ 2 & 2\end{vmatrix}=1\cdot(2)-(-1)\cdot(-7)+3\cdot(-6)=-23;$$

$$\begin{vmatrix}1 & -1 & 2\\ -1 & 2 & 1\\ 2 & 2 & 8\end{vmatrix}=1\cdot\begin{vmatrix}2 & 1\\ 2 & 8\end{vmatrix}-(-1)\cdot\begin{vmatrix}-1 & 1\\ 2 & 8\end{vmatrix}+2\cdot\begin{vmatrix}-1 & 2\\ 2 & 2\end{vmatrix}=1\cdot(14)-(-1)\cdot(-10)+2\cdot(-6)=-8.$$

O determinante original fica: $3\cdot(51)-0+7\cdot(-23)-6\cdot(-8)=40$.

2. a. 0, pois a terceira linha é múltiplo da primeira.

2. b. $3\cdot(-2)\cdot 4=-24$, pois trata-se de uma matriz triangular superior.

2. c. $\begin{vmatrix}0 & 0 & 1\\ 0 & 5 & 2\\ 3 & -1 & 4\end{vmatrix}=-\begin{vmatrix}3 & -1 & 4\\ 0 & 5 & 2\\ 0 & 0 & 1\end{vmatrix}=-15$, permutando, no determinante original, a primeira linha com a terceira.

2. d. 0, pois a terceira coluna é igual a -2 vezes a primeira coluna.

2. e. 0, pois a terceira linha é a soma da primeira e a segunda.

2. f. 0, pois a primeira coluna é a soma da segunda e a terceira.

2. g. $\begin{vmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{vmatrix}=-\begin{vmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{vmatrix}=-1$, permutando a segunda e terceira linhas.

2. h. $\begin{vmatrix}0 & 2 & 0 & 0\\ -3 & 0 & 0 & 0\\ 0 & 0 & 0 & 4\\ 0 & 0 & 1 & 0\end{vmatrix}=-\left(-\begin{vmatrix}-3 & 0 & 0 & 0\\ 0 & 2 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 4\end{vmatrix}\right)=-24$, permutando a primeira com a segunda linhas, e a terceira com a quarta linhas (muda o sinal duas vezes).

2. i. 0, porque podemos fazer somar à linha 1 a linha 2, e à linha 3 a linha 4:

$\begin{vmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ 1 & 1 & 0 & 0\\ 0 & 0 & 1 & 1\end{vmatrix}=\begin{vmatrix}1 & 1 & 1 & 1\\ 0 & 1 & 0 & 1\\ 1 & 1 & 1 & 1\\ 0 & 0 & 1 & 1\end{vmatrix}=0$, observando que no último determinante as linhas 1 e 3 são iguais.