Mecânica Estatística Quântica – 2

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Estatística de Fermi-Dirac

$$\ln \Xi = \sum_i \ln\left(1 + e^{-\beta(\varepsilon_i - \mu)}\right) \quad \rightarrow \quad \sum_i \rightarrow \sum_s \sum_{\vec{k}} \rightarrow g_s \frac{V}{(2\pi)^3} \int d^3k$$

g_s é o número de estados de spin para cada valor de $\vec{k}$: **degenerescência de spin**

"se cada partícula tem um momento angular de spin intrínseco s, as possíveis orientações deste spin são dadas por sua projeção

$$m_s = -s, -s+1, \ldots, s-1, s$$

ou seja, existem $g_s = 2s+1$ possíveis estados de mesma energia associados com cada estado de translação de uma partícula."

$$\left.\begin{aligned} \frac{p}{k_B T} &= \frac{g_s}{\lambda^3} f_{5/2}(z) \\ \frac{N}{V} &= \frac{g_s}{\lambda^3} f_{3/2}(z) \end{aligned}\right\} \qquad \frac{U}{V} = \frac{3}{2} k_B T \frac{g_s}{\lambda^3} f_{5/2}(z) \quad \rightarrow \quad U = \frac{3}{2} N k_B T \frac{f_{5/2}(z)}{f_{3/2}(z)}$$

Estatística de Fermi-Dirac

Calor específico

$$\frac{C_V}{Nk_B} = \frac{1}{Nk_B}\left(\frac{\partial U}{\partial T}\right)_{V,N} \qquad \left(U = \frac{3}{2}Nk_BT\frac{f_{5/2}(z)}{f_{3/2}(z)}\right)$$

$$= \frac{1}{Nk_B}\left[\frac{3}{2}Nk_B\frac{f_{5/2}(z)}{f_{3/2}(z)} + \frac{3}{2}\frac{Nk_BT}{f_{3/2}(z)}\left(\frac{\partial f_{5/2}(z)}{\partial T}\right)_{V,N} - \frac{3}{2}Nk_BT\frac{f_{5/2}(z)}{(f_{3/2}(z))^2}\left(\frac{\partial f_{3/2}(z)}{\partial T}\right)_{V,N}\right]$$

$$\frac{N}{V} = \frac{g_s}{\lambda^3}f_{3/2}(z) \quad \rightarrow \quad f_{3/2}(z) = \frac{\lambda^3}{vg_s} \qquad \left(v = \frac{V}{N}\right)$$

$$\left(\frac{\partial f_{3/2}(z)}{\partial T}\right)_{V,N} = \left(\frac{\partial f_{3/2}(z)}{\partial T}\right)_v = \frac{1}{g_sv}3\lambda^2\frac{d\lambda}{dT} \qquad \left(\lambda = \frac{h}{\sqrt{2\pi mk_BT}}\right)$$

$$= -\frac{1}{g_sv}\frac{3}{2}\lambda^2\frac{h}{(2\pi mk_BT)^{3/2}}\frac{(2\pi mk_BT)}{T} = -\frac{1}{g_sv}\frac{3}{2}\frac{\lambda^3}{T} = -\frac{3}{2T}f_{3/2}(z)$$

Estatística de Fermi-Dirac

Calor específico

$$\frac{C_V}{Nk_B} = \frac{1}{Nk_B}\left(\frac{\partial U}{\partial T}\right)_{V,N} \qquad \left(U = \frac{3}{2}Nk_B T\frac{f_{5/2}(z)}{f_{3/2}(z)}\right)$$

$$= \frac{1}{Nk_B}\left[\frac{3}{2}Nk_B\frac{f_{5/2}(z)}{f_{3/2}(z)} + \frac{3}{2}\frac{Nk_B T}{f_{3/2}(z)}\left(\frac{\partial f_{5/2}(z)}{\partial T}\right)_{V,N} - \frac{3}{2}Nk_B T\frac{f_{5/2}(z)}{(f_{3/2}(z))^2}\left(\frac{\partial f_{3/2}(z)}{\partial T}\right)_{V,N}\right]$$

$$z\frac{\partial f_\nu(z)}{\partial z} = f_{\nu-1}(z) \quad \rightarrow \quad \left(\frac{\partial f_{5/2}(z)}{\partial T}\right)_{V,N} = \left(\frac{\partial f_{5/2}(z)}{\partial z}\right)\left(\frac{dz}{dT}\right)_{V,N} = f_{3/2}(z)\,\frac{1}{z}\left(\frac{dz}{dT}\right)_{V,N}$$

$$\left(\frac{\partial f_{3/2}(z)}{\partial T}\right)_{V,N} = -\frac{3}{2T}f_{3/2}(z) \quad \rightarrow \quad \frac{\partial f_{3/2}(z)}{\partial z}\left(\frac{dz}{dT}\right)_{V,N} = -\frac{3}{2T}f_{3/2}(z)$$

$$f_{1/2}(z)\frac{1}{z}\left(\frac{dz}{dT}\right)_{V,N} = -\frac{3}{2T}f_{3/2}(z) \quad \rightarrow \quad \frac{1}{z}\left(\frac{dz}{dT}\right)_{V,N} = -\frac{3}{2T}\frac{f_{3/2}(z)}{f_{1/2}(z)}$$

Estatística de Fermi-Dirac

Calor específico

$$\frac{C_V}{Nk_B} = \frac{1}{Nk_B}\left(\frac{\partial U}{\partial T}\right)_{V,N} \qquad \left(U = \frac{3}{2}Nk_B T\frac{f_{5/2}(z)}{f_{3/2}(z)}\right)$$

$$= \frac{1}{Nk_B}\left[\frac{3}{2}Nk_B\frac{f_{5/2}(z)}{f_{3/2}(z)} + \frac{3}{2}\frac{Nk_B T}{f_{3/2}(z)}\left(\frac{\partial f_{5/2}(z)}{\partial T}\right)_{V,N} - \frac{3}{2}Nk_B T\frac{f_{5/2}(z)}{(f_{3/2}(z))^2}\left(\frac{\partial f_{3/2}(z)}{\partial T}\right)_{V,N}\right]$$

$$\left(\frac{\partial f_{5/2}(z)}{\partial T}\right)_{V,N} = -\frac{3}{2T}\frac{[f_{3/2}(z)]^2}{f_{1/2}(z)} \qquad \left(\frac{\partial f_{3/2}(z)}{\partial T}\right)_{V,N} = -\frac{3}{2T}f_{3/2}(z)$$

$$\frac{C_V}{Nk_B} = \frac{3}{2}\left[\frac{f_{5/2}(z)}{f_{3/2}(z)} - \frac{T}{f_{3/2}(z)}\frac{3}{2T}\frac{[f_{3/2}(z)]^2}{f_{1/2}(z)} + T\frac{f_{5/2}(z)}{[f_{3/2}(z)]^2}\frac{3}{2T}f_{3/2}(z)\right]$$

$$= \frac{3}{2}\left[\frac{f_{5/2}(z)}{f_{3/2}(z)} - \frac{3}{2}\frac{f_{3/2}(z)}{f_{1/2}(z)} + \frac{3}{2}\frac{f_{5/2}(z)}{f_{3/2}(z)}\right] \quad \rightarrow \quad \boxed{\frac{C_V}{Nk_B} = \frac{15}{4}\frac{f_{5/2}(z)}{f_{3/2}(z)} - \frac{9}{4}\frac{f_{3/2}(z)}{f_{1/2}(z)}}$$

Estatística de Fermi-Dirac

Potencial de Helmholtz

$$F = G - pV \quad \text{sistemas com uma componente} \rightarrow G = Ng = N\mu \quad \left(z = e^{\beta\mu}\right)$$

$$= Nk_BT\ln z - pV \rightarrow pV = Nk_BT\frac{f_{5/2}(z)}{f_{3/2}(z)} \rightarrow \boxed{F = Nk_BT\left[\ln z - \frac{f_{5/2}(z)}{f_{3/2}(z)}\right]}$$

Entropia $S = \frac{U}{T} - \frac{F}{T}$

$$\begin{cases} \frac{U}{V} = \frac{3}{2}k_BT\frac{g_s}{\lambda^3}f_{5/2}(z) \\ F = Nk_BT\left[\ln z - \frac{f_{5/2}(z)}{f_{3/2}(z)}\right] \end{cases}$$

$$S = \frac{3}{2}k_BV\frac{g_s}{\lambda^3}f_{5/2}(z) - Nk_B\ln z + Nk_B\frac{f_{5/2}(z)}{f_{3/2}(z)} \quad \left(\frac{N}{V} = \frac{g_s}{\lambda^3}f_{3/2}(z)\right)$$

$$= \frac{3}{2}k_B\frac{\lambda^3N}{g_sf_{3/2}(z)}\frac{g_s}{\lambda^3}f_{5/2}(z) - Nk_B\ln z + Nk_B\frac{f_{5/2}(z)}{f_{3/2}(z)} \quad \boxed{S = Nk_B\left[\frac{5}{2}\frac{f_{5/2}(z)}{f_{3/2}(z)} - \ln z\right]}$$

Estatística de Fermi-Dirac

Resumo:

$$\left.\begin{aligned}\frac{p}{k_B T} &= \frac{g_s}{\lambda^3} f_{5/2}(z) \\ \frac{N}{V} &= \frac{g_s}{\lambda^3} f_{3/2}(z)\end{aligned}\right\} \qquad U = \frac{3}{2} N k_B T \frac{f_{5/2}(z)}{f_{3/2}(z)}$$

$$\frac{C_V}{N k_B} = \frac{15}{4}\frac{f_{5/2}(z)}{f_{3/2}(z)} - \frac{9}{4}\frac{f_{3/2}(z)}{f_{1/2}(z)}$$

$$F = N k_B T \left[\ln z - \frac{f_{5/2}(z)}{f_{3/2}(z)}\right] \qquad S = N k_B \left[\frac{5}{2}\frac{f_{5/2}(z)}{f_{3/2}(z)} - \ln z\right]$$

$$\left.\begin{aligned}f_{5/2}(z) &= \frac{4}{3\sqrt{\pi}} \int_0^\infty \frac{x^{3/2}}{z^{-1}e^x+1}\,dx \\ f_{3/2}(z) &= \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{x^{1/2}}{z^{-1}e^x+1}\,dx \\ f_{1/2}(z) &= \frac{1}{\sqrt{\pi}} \int_0^\infty \frac{x^{-1/2}}{z^{-1}e^x+1}\,dx\end{aligned}\right\}$$

As propriedades do gás de Fermi são definidas pelo comportamento da função de Fermi $f_\nu(z)$.

Esta função deve ser calculada numericamente, exceto nos casos limites em que z é pequeno.

Estatística de Fermi-Dirac

Limite de z pequeno

$$f_\nu(z) = \frac{1}{\Gamma(\nu)} \int_0^\infty \frac{x^{\nu-1}}{z^{-1}e^x + 1}\, dx = \frac{1}{\Gamma(\nu)} \int_0^\infty x^{\nu-1} \frac{ze^{-x}}{1+ze^{-x}}\, dx$$

Se z **é pequeno** podemos escrever

$$-\frac{1}{1+ze^{-x}} = \sum_{l=0}^{\infty}(-1)^{l-1}\,(ze^{-x})^l = -1 + \sum_{l=1}^{\infty}(-1)^{l-1}\,(ze^{-x})^l \quad \rightarrow \quad 1 - \frac{1}{1+ze^{-x}} = \sum_{l=1}^{\infty}(-1)^{l-1}\,(ze^{-x})^l$$

$$\frac{ze^{-x}}{1+ze^{-x}} = \sum_{l=1}^{\infty}(-1)^{l-1}\,(ze^{-x})^l$$

tal que

$$f_\nu(z) = \frac{1}{\Gamma(\nu)} \int_0^\infty x^{\nu-1} \sum_{l=1}^{\infty}(-1)^{l-1}\,(ze^{-x})^l\, dx = \frac{1}{\Gamma(\nu)} \sum_{l=1}^{\infty}(-1)^{l-1}\, z^l \int_0^\infty x^{\nu-1} e^{-lx}\, dx$$

Estatística de Fermi-Dirac

Limite de z pequeno

$$f_\nu(z) = \frac{1}{\Gamma(\nu)} \sum_{l=1}^{\infty} (-1)^{l-1} z^l \int_0^\infty x^{\nu-1} e^{-lx}\, dx$$

troca de variável: $\rightarrow \quad lx = y \quad \left(x = \frac{y}{l}\right) \quad \rightarrow \quad dx = \frac{1}{l} dy$

$$f_\nu(z) = \frac{1}{\Gamma(\nu)} \sum_{l=1}^{\infty} (-1)^{l-1} z^l \left(\frac{1}{l}\right)^{\nu-1} \frac{1}{l} \underbrace{\int_0^\infty y^{\nu-1} e^{-y}\, dy}_{\equiv \Gamma(\nu)} \rightarrow \boxed{f_\nu(z) = \sum_{l=1}^{\infty} (-1)^{l-1} \frac{z^l}{l^\nu}}$$

Limite de $z \ll 1$ (regime clássico)

$$f_\nu(z) = \sum_{l=1}^{\infty} (-1)^{l-1} \frac{z^l}{l^\nu} \approx z - \frac{z^2}{2^\nu} + \frac{z^3}{3^\nu} + \ldots \rightarrow \boxed{f_\nu(z) = z} \quad \begin{cases} pV = Nk_BT \\ U = \frac{3}{2} Nk_BT \\ C_V = \frac{3}{2} Nk_B \end{cases}$$

Estatística de Fermi-Dirac

Limite de z pequeno (mas não muito menor do que a unidade)

$$f_\nu(z) = \sum_{l=1}^{\infty} (-1)^{l-1} \frac{z^l}{l^\nu} \qquad \begin{cases} \dfrac{p}{k_B T} = \dfrac{g_s}{\lambda^3} f_{5/2}(z) = \dfrac{g_s}{\lambda^3} \displaystyle\sum_{l=1}^{\infty} (-1)^{l-1} \frac{z^l}{l^{5/2}} \\[2ex] \dfrac{N}{V} = \dfrac{g_s}{\lambda^3} f_{3/2}(z) = \dfrac{g_s}{\lambda^3} \displaystyle\sum_{l=1}^{\infty} (-1)^{l-1} \frac{z^l}{l^{3/2}} \end{cases}$$

$$\frac{p}{k_B T} \frac{V}{N} = \frac{g_s V}{\lambda^3 N} \sum_{l=1}^{\infty} (-1)^{l-1} \frac{z^l}{l^{5/2}} \qquad \frac{N}{V} = \frac{g_s}{\lambda^3} f_{3/2}(z) \underbrace{\longrightarrow}_{z \ll 1} \frac{N}{V} = \frac{g_s}{\lambda^3} z \quad \rightarrow \quad z = \left(\frac{\lambda^3}{g_s v} \right)$$

Proposta: **Equação de estado** na forma de uma **expansão do virial** (em potências de z)

$$\frac{pV}{N k_B T} = \sum_{l=1}^{\infty} (-1)^{l-1} a_l \left(\frac{\lambda^3}{g_s v} \right)^{l-1} \qquad a_l \rightarrow \text{coeficientes do virial}$$

Os coeficientes do virial a_l são obtidos das expansões de $p/k_B T$ e N/V em termos de z

Estatística de Fermi-Dirac

Limite de z pequeno (mas não muito menor do que a unidade)

$$\frac{pV}{Nk_BT} = \frac{p/k_BT}{N/V} = \sum_{l=1}^{\infty}(-1)^{l-1}\,a_l\left(\frac{\lambda^3}{g_s v}\right)^{l-1}$$

$$\begin{cases} \dfrac{p}{k_BT} = \dfrac{g_s}{\lambda^3}\displaystyle\sum_{l=1}^{\infty}(-1)^{l-1}\frac{z^l}{l^{5/2}} \\ \dfrac{N}{V} = \dfrac{g_s}{\lambda^3}\displaystyle\sum_{l=1}^{\infty}(-1)^{l-1}\frac{z^l}{l^{3/2}} \end{cases}$$

$$\frac{\sum_{l=1}^{\infty}(-1)^{l-1}\frac{z^l}{l^{5/2}}}{\sum_{l=1}^{\infty}(-1)^{l-1}\frac{z^l}{l^{3/2}}} = \sum_{l=1}^{\infty}(-1)^{l-1}\,a_l\left(\frac{\lambda^3}{g_s v}\right)^{l-1}$$

$$\left(\frac{\lambda^3}{g_s v}\right) = \sum_{l=1}^{\infty}(-1)^{l-1}\frac{z^l}{l^{3/2}}$$

$$\frac{\sum_{l=1}^{\infty}(-1)^{l-1}\frac{z^l}{l^{5/2}}}{\sum_{l=1}^{\infty}(-1)^{l-1}\frac{z^l}{l^{3/2}}} = \sum_{l=1}^{\infty}(-1)^{l-1}\,a_l\left(\sum_{l=1}^{\infty}(-1)^{l-1}\frac{z^l}{l^{3/2}}\right)^{l-1}$$

Estatística de Fermi-Dirac

Limite de z pequeno (mas não muito menor do que a unidade)

$$\frac{p/k_BT}{N/V} = \sum_{l=1}^{\infty}(-1)^{l-1}\,a_l\left(\frac{\lambda^3}{g_s v}\right)^{l-1}$$

$$\begin{cases} \dfrac{p}{k_BT} = \dfrac{g_s}{\lambda^3}\displaystyle\sum_{l=1}^{\infty}(-1)^{l-1}\frac{z^l}{l^{5/2}} \\ \dfrac{N}{V} = \dfrac{g_s}{\lambda^3}\displaystyle\sum_{l=1}^{\infty}(-1)^{l-1}\frac{z^l}{l^{3/2}} \end{cases}$$

$$\frac{\sum_{l=1}^{\infty}(-1)^{l-1}\frac{z^l}{l^{5/2}}}{\sum_{l=1}^{\infty}(-1)^{l-1}\frac{z^l}{l^{3/2}}} = \sum_{l=1}^{\infty}(-1)^{l-1}\,a_l\left(\sum_{l=1}^{\infty}(-1)^{l-1}\frac{z^l}{l^{3/2}}\right)^{l-1}$$

$$\frac{z-\frac{z^2}{2^{5/2}}+\frac{z^3}{3^{5/2}}+\ldots}{z-\frac{z^2}{2^{3/2}}+\frac{z^3}{3^{3/2}}+\ldots} = a_1 - a_2\left(z-\frac{z^2}{2^{3/2}}+\frac{z^3}{3^{3/2}}+\ldots\right)+a_3\left(z-\frac{z^2}{2^{3/2}}+\frac{z^3}{3^{3/2}}+\ldots\right)^2+\ldots$$

Estatística de Fermi-Dirac

Limite de z pequeno (mas não muito menor do que a unidade)

$$\frac{p/k_BT}{N/V} = \sum_{l=1}^{\infty}(-1)^{l-1}\, a_l \left(\frac{\lambda^3}{g_s v}\right)^{l-1}$$

$$\begin{cases} \dfrac{p}{k_BT} = \dfrac{g_s}{\lambda^3}\displaystyle\sum_{l=1}^{\infty}(-1)^{l-1}\frac{z^l}{l^{5/2}} \\ \dfrac{N}{V} = \dfrac{g_s}{\lambda^3}\displaystyle\sum_{l=1}^{\infty}(-1)^{l-1}\frac{z^l}{l^{3/2}} \end{cases}$$

$$\frac{\sum_{l=1}^{\infty}(-1)^{l-1}\frac{z^l}{l^{5/2}}}{\sum_{l=1}^{\infty}(-1)^{l-1}\frac{z^l}{l^{3/2}}} = \sum_{l=1}^{\infty}(-1)^{l-1}\, a_l \left(\sum_{l=1}^{\infty}(-1)^{l-1}\frac{z^l}{l^{3/2}}\right)^{l-1}$$

$$z - \frac{z^2}{2^{5/2}} + \frac{z^3}{3^{5/2}} + \ldots = a_1\left(z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} + \ldots\right) - a_2\left(z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} + \ldots\right)^2 + a_3\left(z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} + \ldots\right)^3 + \ldots$$

Estatística de Fermi-Dirac

Limite de z pequeno (mas não muito menor do que a unidade)

$$\frac{p/k_BT}{N/V} = \sum_{l=1}^{\infty}(-1)^{l-1}\, a_l \left(\frac{\lambda^3}{g_s v}\right)^{l-1}$$

$$z - \frac{z^2}{2^{5/2}} + \frac{z^3}{3^{5/2}} + \ldots = a_1\left(z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} + \ldots\right) - a_2\left(z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} + \ldots\right)^2 + a_3\left(z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} + \ldots\right)^3 + \ldots$$

Os valores de a_l são obtidos igualando os coeficientes de z^l de ambos os lados da igualdade:

$$z^1 \to a_1 = 1 \qquad z^2 \to -\frac{1}{2^{5/2}} = -a_1\frac{1}{2^{3/2}} - a_2 \qquad a_2 = -\frac{1}{4\sqrt{2}} \approx -0.17678$$

$$z^3 \to \frac{1}{3^{5/2}} = a_1\frac{1}{3^{3/2}} + 2a_2\frac{1}{2^{3/2}} + a_3 \qquad a_3 = -\frac{2}{9\sqrt{3}} + \frac{1}{8} \approx -0.0033$$

Estatística de Fermi-Dirac

Limite de z pequeno (mas não muito menor do que a unidade)

$$\frac{p/k_B T}{N/V} = \sum_{l=1}^{\infty} (-1)^{l-1} a_l \left(\frac{\lambda^3}{g_s v} \right)^{l-1}$$

Equação de estado na forma de uma **expansão do virial**

$$pV = N k_B T \left[1 + 0.17678 \left(\frac{\lambda^3}{g_s v} \right) - 0.0033 \left(\frac{\lambda^3}{g_s v} \right)^2 + \ldots \right]$$

$$\left(\frac{\lambda^3}{g_s v} \right) \rightarrow \begin{cases} \text{correções quânticas à equação de estado de gás ideal clássico} \\ \text{aumenta a pressão do gás de Fermi} \\ \text{resultado da repulsão produzida pelo princípio de exclusão de Pauli} \end{cases}$$

Estatística de Fermi-Dirac

Limite de z pequeno (mas não muito menor do que a unidade)

Calor específico $\rightarrow$ $\dfrac{C_V}{Nk_B} = \dfrac{1}{Nk_B}\left(\dfrac{\partial U}{\partial T}\right)_{V,N}$ $\begin{cases} p = \dfrac{2}{3}\dfrac{U}{V} \\ \dfrac{pV}{Nk_B} = T\displaystyle\sum_{l=1}^{\infty}(-1)^{l-1}a_l\left(\dfrac{\lambda^3}{g_s v}\right)^{l-1} \end{cases}$

$$\frac{C_V}{Nk_B} = \frac{3}{2}\left[\frac{\partial}{\partial T}\left(\frac{pV}{Nk_B}\right)\right]_{V,N} = \frac{3}{2}\left[\sum_{l=1}^{\infty}(-1)^{l-1}a_l\left(\frac{\lambda^3}{g_s v}\right)^{l-1} + T\sum_{l=1}^{\infty}(-1)^{l-1}a_l\,(l-1)\left(\frac{\lambda^3}{g_s v}\right)^{l-2}\frac{3\lambda^2}{g_s v}\frac{d\lambda}{dT}\right]$$

$$\text{como} \quad \lambda = \frac{h}{\sqrt{2\pi m k_B T}} \quad \rightarrow \quad \frac{d\lambda}{dT} = -\frac{\lambda}{2T}$$

$$\begin{aligned} \frac{C_V}{Nk_B} &= \frac{3}{2}\left[\sum_{l=1}^{\infty}(-1)^{l-1}a_l\left(\frac{\lambda^3}{g_s v}\right)^{l-1} - \frac{3}{2}\sum_{l=1}^{\infty}(-1)^{l-1}a_l\,(l-1)\left(\frac{\lambda^3}{g_s v}\right)^{l-1}\right] \\ &= \frac{3}{2}\sum_{l=1}^{\infty}(-1)^{l-1}\frac{5-3l}{2}\,a_l\left(\frac{\lambda^3}{g_s v}\right)^{l-1} \end{aligned}$$

Estatística de Fermi-Dirac

Limite de z pequeno (mas não muito menor do que a unidade)

Calor específico $\rightarrow$ $\frac{C_V}{Nk_B} = \frac{1}{Nk_B}\left(\frac{\partial U}{\partial T}\right)_{V,N}$

$$\begin{cases} p = \frac{2}{3}\frac{U}{V} \\ \frac{pV}{Nk_B} = T\sum_{l=1}^{\infty}(-1)^{l-1}\, a_l \left(\frac{\lambda^3}{g_s v}\right)^{l-1} \end{cases}$$

$$\frac{C_V}{Nk_B} = \frac{3}{2}\sum_{l=1}^{\infty}(-1)^{l-1}\,\frac{5-3l}{2}\,a_l \left(\frac{\lambda^3}{g_s v}\right)^{l-1} \qquad \begin{cases} a_1 = 1 \\ a_2 \approx -0.17678 \\ a_3 \approx -0.0033 \end{cases}$$

$$\boxed{\frac{C_V}{Nk_B} = \frac{3}{2}\left[1 - 0.0884\left(\frac{\lambda^3}{g_s v}\right) + 0.0066\left(\frac{\lambda^3}{g_s v}\right)^2 - 0.0004\left(\frac{\lambda^3}{g_s v}\right)^3 + \ldots\right]}$$

Para temperaturas finitas, o calor específico do gás de Fermi é menor do que o limite clássico $3k_B/2$

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Limite de z pequeno (mas não muito menor do que a unidade)

$$\frac{C_V}{Nk_B} = \frac{3}{2}\left[1 - 0.0884\left(\frac{\lambda^3}{g_s v}\right) + 0.0066\left(\frac{\lambda^3}{g_s v}\right)^2 - 0.0004\left(\frac{\lambda^3}{g_s v}\right)^3 + \ldots\right]$$

Para temperaturas finitas, o calor específico do gás de Fermi é menor do que o limite clássico $3k_B/2$