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The Reverend Thomas Bayes, FRS: A Biography to Celebrate the Tercentenary of His Birth

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Abstract. Thomas Bayes, from whom Bayes theorem takes its name, was probably born in 1701, so the year 2001 marked the 300th anniversary of his birth. This biography was written to celebrate this anniversary. The current sketch of his life includes his family background and education, as well as his scientific and theological work. In contrast to some, but not all, biographies of Bayes, the current biography is an attempt to cover areas beyond Bayes' scientific work. When commenting on the writing of scientific biography, Pearson [(1978). *The History of Statistics in the 17th and 18th Centuries*. . . . Charles Griffin and Company, London] stated, "it is impossible to understand a man's work unless you understand something of his character and unless you understand something of his environment. And his environment means the state of affairs social and political of his own age." The intention here is to follow this general approach to biography.

There is very little primary source material on Bayes and his work. For example, only three of his letters and a notebook containing some sketches of his own work, almost all unpublished, as well as notes on the work of others are known to have survived. Neither the letters nor the notebook is dated, and only one of the letters can be dated accurately from internal evidence. This biography contains new information about Bayes. In particular, among the papers of the 2nd Earl Stanhope, letters and papers of Bayes have been uncovered that previously were not known to exist. The letters indirectly confirm the centrality of Stanhope in Bayes' election to the Royal Society. They also provide evidence that Bayes was part of a network of mathematicians initially centered on Stanhope. In addition, the letters shed light on Bayes' work in infinite series.

1. INTRODUCTION

The year 2001 probably marked the 300th anniversary of the birth of Thomas Bayes (1701?–1761). This biography was written in celebration of that anniversary and an abbreviated version of it was presented at the Seoul, Korea, meetings of the International Statistical Institute in 2001. There are already several biographies or biographical sketches of Bayes: a 19th century biography appeared in Fisher (1857) and some 20th century biographies are given in Anderson (1941), Barnard (1958), Dale (1991a), Edwards (1993), Hacking (1970–1980), Holland (1962) and Pearson (1978). When commenting on the writing of scientific biography, Pearson (1978), in lectures given during the 1920s and early 1930s, stated:

> . . . it is impossible to understand a man's work unless you understand something of his character and unless you understand something of his environment. And his environment means the state of affairs social and political of his own age.

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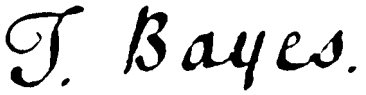


FIG. 1. *Signature of Thomas Bayes from a letter in the Centre for Kentish Studies.*

In their own biographies of Bayes, both Holland (1962) and Pearson (1978) took this statement to heart. My intention is to follow this general approach to biography while including new information on Bayes that was not available to either of these two authors.

To maintain the flow of the text, I will present the complete biography with little discussion of other biographies of Bayes that have been written. The current biography relies on all known source material related directly to Bayes, including new material recently discovered in the Stanhope papers held at the Centre for Kentish Studies. (Bayes' signature is shown in Figure 1.) Since there is very little primary source material on Bayes, filling in the details in some areas of his life requires some conjecture. I will make explicit where conjectures are made and support them with evidence both from sources contemporary to Bayes or from later authors. There are some differences between this biography and others about Bayes that have been written. Some of the more important differences will be noted in the text.

2. RELIGIOUS BACKGROUND

Thomas Bayes was a Presbyterian minister. To understand the context in which Bayes lived and worked, it is first necessary to know something of English religious dissent or nonconformity in the late 17th and early 18th centuries. For our purposes, the central issue was the use of the *Book of Common Prayer* that Thomas Cranmer, Archbishop of Canterbury, completed in 1549 for use in the Church of England. After Mary took the throne in 1553, Cranmer was burned at the stake for heresy and the *Book of Common Prayer* was officially abandoned. In 1559, one year after Elizabeth I ascended the throne, the prayer book was reinstated and an Act of Uniformity was passed that required all churches in England to use the *Book of Common Prayer* for worship services. This continued until the period of the Commonwealth (1649–1660) when the monarchy was overthrown. At that time, congregations were allowed to use their own forms of worship. Two years after the restoration of the monarchy in 1660, another Act of Uniformity was passed that again required the use of the *Book of Common Prayer* for worship services. About 2000 clergy refused to conform to the terms of the Act and were ejected from their positions. These clergymen were often taken on as private chaplains by local gentry, and so were able to continue preaching and ministering to their followers. The term "Nonconformist" or "Dissenting" minister takes its name from the events stemming from the later Act of Uniformity. In the year following the 1688 revolution that put William and Mary on the throne of England, an Act of Toleration was passed that allowed Protestant Nonconformists or Dissenters freedom of worship. One condition was that nonconformist meetinghouses, or chapels as they were subsequently known, had to be licensed for worship. This freedom of worship was not extended to Roman Catholics. Furthermore, all Nonconformists were banned from holding any public office and were excluded from the universities. As a result of this ban, Nonconformist ministers developed their own institutions, called *Dissenting academies*, for the education of those wishing to enter the ministry and for sons of members of Nonconformist or Dissenting congregations. The best of these academies rivaled the universities in terms of the quality of education. Sons of Nonconformist parents were also sent to Scottish universities, where the ban was not in effect, or to Holland, usually the University of Leiden.

Religious nonconformity or dissent might take its name from an event in 1660, but its theological and institutional character grew out of the Puritan movement of the 16th century. By the late 17th century there were four distinct groups that had evolved from Puritan origins: Presbyterians, Congregationalists (or Independents), Baptists and Quakers. The first three were known collectively as *The Three Denominations* since there was a certain amount of official cooperation among them in the late 17th century and well into the 18th century, especially regarding their civil rights. The defining differences between the three denominations were organizational and doctrinal. All three rejected the episcopacy. The Presbyterians favored a hierarchy of church government that centered on a general assembly. Rather than a hierarchical government, the Congregationalists wanted independence for each congregation. In the 18th century, the Presbyterians and Congregationalists drifted apart on matters of doctrine, in particular on the doctrine of grace and on the spread of Unitarian ideas especially among the Presbyterians. The Baptists were distinguished from the other two groups by their rejection of infant baptism.

3. FAMILY BACKGROUND

Thomas Bayes came from a prominent Nonconformist family of Sheffield in the north of England. Well before Thomas was born, the family had made its fortune in the cutlery industry for which the town was famous. Pedigrees with some biographical information for the Bayes family and the allied families of Cotton and Wordsworth can be found in Clay (1894–1895), and biographical material relating to the family's time in Sheffield can be found in Leader (1897) and Leader (1905–1906). Some of the Bayes family connections found in Clay (1894–1895) and through wills in the Bayes family are produced in Figure 2.

Richard Bayes (1596–1677), a great-grandfather of Thomas Bayes, was a successful cutler in Sheffield. In 1643, Richard Bayes served in the rotating position of Master of the Company of Cutlers of Hallamshire. The company, under its act of incorporation, had authority over the cutlery industry for the area in and around Sheffield. Richard was sufficiently well off that he sent one of his sons, Samuel Bayes (1635–1681), to Trinity College Cambridge during the Commonwealth period; Samuel obtained his degree in 1656. Samuel became the vicar of Grendon in Northamptonshire. Another son, Joshua Bayes (1638–1703) followed in his father's footsteps in the cutlery industry, also serving as Master of the Company in 1679. By the second generation there is considerable evidence of wealth, influence and religious nonconformity. Evidence of Joshua Bayes' wealth comes from the size of the house, the fact that he employed a servant and the amount of taxes that he paid. His influence may be taken from his activities in the town government. The two brothers were prominent Nonconformists. Following the 1662 Act of Uniformity, Samuel Bayes was ejected from his parish, eventually living in Manchester (Matthews, 1934). Joshua Bayes was closely involved in the erection of one Nonconformist chapel in Sheffield and had two sons-in-law involved in another Sheffield chapel. One of the sons-in-law, Elias Wordsworth (d. 1723),

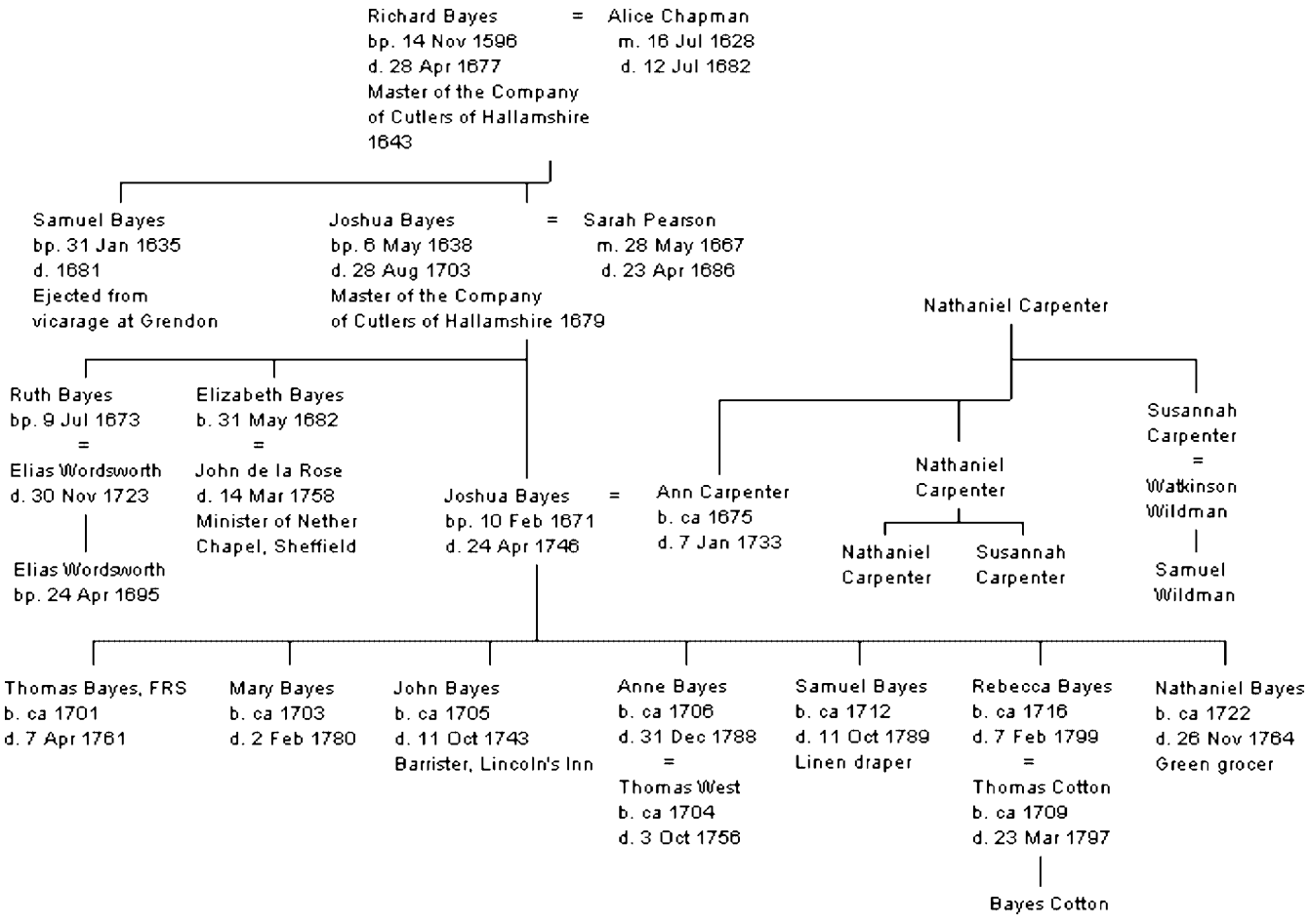


FIG. 2. *Bayes family connections.*

FIG. 3. *Joshua Bayes* (*1671–1746*).

was the founder of this second chapel and the other, John de la Rose, was the minister in it.

The second son of Joshua Bayes (1638–1703) was another Joshua Bayes (1671–1746). Biographical details for the younger Joshua can be found in Wilson (1808–1814, Vol. 4) and *Dictionary of National Biography* (Stephen and Lee, 1921–1922). In 1686, the younger Joshua Bayes (see Figure 3) entered a Dissenting academy where he studied philosophy and divinity. Located in the north of England, this academy had been established by Richard Frankland, who had been ejected from a curacy in the Church of England in 1662 (Matthews, 1934). Due to official government-sanctioned religious persecution, the academy changed locations at least four times over three different counties. At least one of these locations was near Sheffield. Bayes left the Frankland academy in 1694 and went to London where he was among the first group of Presbyterians in London to be ordained publicly to the ministry. In 1706, Bayes became the assistant minister at St. Thomas' Street Chapel in the London suburb of Southwark. He also served as an assistant at the Leather Lane Chapel in Hatton Garden, London. On the death of the minister at the Leather Lane Chapel in 1723, Bayes became the minister there. Joshua Bayes was well respected, both as a preacher and as a man of learning.

Joshua Bayes and his wife Anne née Carpenter were married some time, probably within days, after their marriage license was issued on October 23, 1700. There is no known surviving record of their marriage in Church of England registers and so it is likely that they were married in a Nonconformist chapel. At that time a chapel marriage would have been unusual, though not unheard of (Steele, 1973). A chapel marriage would be consistent with the nature of his ordination—both acts, in the way they were carried out, could be seen as the exercise of new or reclaimed religious rights. Joshua and Anne Bayes had seven children. In their order of birth, the children were Thomas (died 1761, aged 59), Mary (died 1780, aged 76), John (died 1743, aged 38), Anne (died 1788, aged 82), Samuel (died 1789, aged 77), Rebecca (died 1799, aged 82) and Nathaniel (died 1764, aged 42). Of the seven Bayes siblings, only Anne and Rebecca had surviving children. Through their mother Anne (Carpenter) Bayes, who was from London, the siblings had some London cousins. Nathaniel and Susannah Carpenter were children of Anne's brother Nathaniel Carpenter, and Samuel Wildman was the son of Anne's sister Susannah and her husband Watkinson Wildman.

None of the birth or baptismal dates for Thomas Bayes or any of his siblings is known. It is likely that all the Bayes children were baptized in Nonconformist chapels, perhaps in the chapel where their father assisted, rather than in the Church of England. None of the registers of the Nonconformist chapels for the time period in which Joshua Bayes ministered at them is known to have survived. The earliest known records for the chapel at Leather Lane date from 1835. The records from St. Thomas Chapel, Southwark, date from 1724, immediately after Joshua Bayes left. This is not unusual for many early Nonconformist congregations. In fact, the currently available records from the chapel in Tunbridge Wells, at which Thomas Bayes was the minister, date from 1830. Many early registers were kept secret, or not at all, for fear of religious discrimination. Another factor was that Nonconformist registers were not considered legal registers of birth. Some Nonconformist families had their children baptized in the chapel only, some in the local parish church and some in both. The Bayes children are probably one example of the first situation. An example of both is Thomas Bayes' cousin Elias Wordsworth, son of Elias Wordsworth (d. 1723); Thomas left this cousin £20 in his will. The younger Elias was baptized on April 24, 1695, in the Nonconformist chapel founded by his father. On the same day he was also baptized according to the Church of England rite in St. Peter's Church in Sheffield. In view of this discussion of Nonconformist records, as well as the date of the parents' marriage and the date of Thomas Bayes' death (April 7, 1761), all that can be said about Bayes' birth date is that it is probably between July of 1701 and April of 1702.

The extended family had a variety of occupations as seen from wills and London directories. The Carpenters

were innkeepers in Friday Street and the Wildmans were goldsmiths in Cheapside. Both Friday Street and Cheapside are near St. Paul's Cathedral in London. In the Bayes family, the eldest son, Thomas, entered the ministry. The next eldest, John Bayes, took up the law. He was admitted to Lincoln's Inn to study law in 1733 and was called to the bar in 1739 (*Records of the Honorable Society of Lincoln's Inn*). The younger sons went into trade: Samuel Bayes was a linen draper operating in Cheapside and his brother Nathaniel was a grocer operating in Snow Hill. By 1759, Samuel Bayes had either gone into partnership with another or sold his business. He eventually moved south of London to Clapham. Among the daughters, Mary never married; Anne Bayes married Thomas West, a London mercer or dealer in textiles; and Rebecca Bayes married Thomas Cotton, an attorney in Hackney. Of interest is a further Presbyterian connection. Thomas Cotton was the son of another Thomas Cotton (1653–1730). The elder Thomas was the Presbyterian minister at Dyott Street in Bloomsbury from 1699 to 1727 (*Dictionary of National Biography*).

The general London society in which Joshua Bayes and, by extension, his family moved may be summed up by Coomer's (1946) description of London Dissenting ministers:

> These were men of high academical attainments, many of them educated in Scottish or Dutch Universities. Some twenty or thirty possessed and adorned the dignity of a Scottish doctorate of divinity. The social circle in which they moved was an elevated one, and peers and peeresses were attendant in their ministrations.

The Bayes family of Thomas Bayes' generation was a wealthy one and close knit in its wealth. When their father Joshua Bayes died in 1746, leaving an estate of £10,000, nearly the entire estate was split among the siblings. As the siblings died one by one, they tended to leave their fortunes to their siblings or to nephews. For example, Nathaniel Bayes received £1600 from his father. He received an additional £400 when his brother Thomas died. He must have been an astute businessman. At his death in 1764 he held in excess of £5000 in assets. Apparently leaving no children, his estate was divided among his surviving siblings, nieces and nephews. Mary Bayes inherited £1800 from her father. At the time of his death, Joshua Bayes was also administering another £600 owned by Mary. Over her lifetime she received more inheritances from her siblings. At her death in 1780 she had an estate of £4000. It was split among her surviving siblings and two nephews. As more siblings died, the bulk of the money tended to go to Bayes Cotton, the son of Rebecca (Bayes) and Thomas Cotton.

The family had made its fortune in Sheffield in the cutlery trade. By the time of Joshua Bayes (1671–1746), the family's wealth was held instead in a variety of investments. Joshua apparently invested in mortgages. His daughter Mary, at her death, held her money in 3% bank annuities. These investments remained intact as they were passed through wills to the next generation. This can be seen from a complex legal dispute played out in the Chancery Court that carried over several years in the mid-18th century. Thomas Bayes and his brother Samuel Bayes, acting as executors of their father's estate, were briefly involved as minor players in this court action in 1749. Circa 1735, Joshua Bayes had lent £1500 to Thomas Gibson and Henry Jacomb at 4% interest; the two borrowers used as collateral a stake they had in a property in Wiltshire that was worth £41,000. The property in Wiltshire was actually collateral received by Gibson and Jacomb on money owed to them. When Joshua Bayes died in 1746, the sons administered the £1500 investment as part of the estate. The original lawsuit was initiated by someone else and had its connection with the Wiltshire property.

4. EARLY LIFE AND EDUCATION

Normal entry to ministry in the Church of England was through a degree taken at Oxford or Cambridge. Since ministry in the Church of England was not of interest to him and neither Oxford nor Cambridge was an option open to him, Thomas Bayes took one of the educational paths open to Nonconformists of his day. He trained for the Presbyterian ministry at the University of Edinburgh, entering that university in 1719 (Dale, 1991a, page 3).

Prior to his studies at Edinburgh, Bayes probably received earlier educational training from John Ward (1679?–1758). The evidence for this assertion is circumstantial. Joshua Bayes and John Ward were friends, at least to the extent that, in 1720, Joshua Bayes received from Ward a copy of a book written by Robert Ainsworth on the antiquities collected by John Kemp, a Fellow of the Royal Society (Ainsworth, 1720). Ward had assisted Ainsworth with the book, providing Ainsworth with descriptions of some parts of the collection as well as other information. Ward briefly

described his assistance in this endeavor to Thomas Bayes in a letter written in Latin dated May 10, 1720. A translation of the relevant section of the letter is:

> Recently I gave your father a book about some principals of ancient things, compiled partly through my own and partly through a friend's efforts. This, I suppose, he will send to you shortly.

In the other direction of the friendship, when Ward wrote his *Lives of the Professors of Gresham College*, Joshua Bayes helped to pay for the printing in advance by being a subscriber to the publication (Ward, 1740). With regard to friendship in general, Birch (1766) says in a biography of Ward:

> He [Ward] continued in his employment in the navy office till the summer of the year 1710, when he thought proper to resign it; and finding no other means of gratifying his zeal for the acquisition of knowledge, was induced to undertake the education of a certain number of the children of his friends; chusing rather, as he expressed himself, to converse even with boys upon subjects of literature than to transact the ordinary affairs of life among men. For this purpose he open'd a school in Tenter Alley in Moor Fields, which he kept for many years.

In the 1720 letter from Ward to Thomas Bayes, much of the content was devoted to advice from Ward on how Bayes could improve his Latin composition, so it is likely that Bayes studied with Ward at his school until he enrolled at the University of Edinburgh. The only piece of evidence that is against Bayes being Ward's student is that a list of Ward's students exists, covering the years 1715–1731. Thomas Bayes does not appear on the list. The list may be incomplete or Bayes may have attended the school only between 1710 and 1715, and then had private tutoring from Ward afterward.

What is also apparent in the letter from Ward to Bayes is that at the time the letter was written Bayes was reading classical authors in both Greek and Latin. According to Timpson (1859), Bayes was the best Greek scholar that Richard Onely had ever met. Onely was the Church of England rector of the parish of Speldhurst near Tunbridge Wells. This is probably an error since Onely did not arrive at Speldhurst until after Bayes' death, and there is no reason to assume that they knew each other before Bayes arrived in Tunbridge Wells. The reference was probably to Bayes' successor at Tunbridge Wells, William Johnston.

Ward was appointed Professor of Rhetoric at Gresham College in 1720 and elected Fellow of the Royal Society in 1723. The college was founded in 1579 by Sir Thomas Gresham for the delivery of lectures that were free to all who cared to attend. The professorships, seven in all, were in the areas of divinity, music, astronomy, geometry, physics, law and rhetoric. The duties of the professor were not onerous—one or two lectures per week. One other stipulation was that professors had to be unmarried and live in college (Weinreb and Hibbert, 1983). In view of Birch's comment that Ward kept his school for many years, he may have taught at this school while he maintained his professorship.

The teaching year at the University of Edinburgh began at various times in the fall. *Scots Magazine* for 1741 gave a general description of the teaching schedule at the University for that year and for many of the courses of lectures, called colleges at the time, that were offered. Presumably the situation at Edinburgh was quite similar two decades earlier. There were no formal terms at the University of Edinburgh; each professor gave one or more colleges over a set period of time. Commencement times for these colleges varied from mid-October to mid-November and the colleges usually ended at various times in May and June. For example, instruction at Divinity Hall began in mid-November of 1741. The mathematics college, given by Colin Maclaurin, the Professor of Mathematics at the time, began on November 1; another professor's college in philosophy started on October 10. Although the teaching year normally began in October or November, Thomas Bayes probably entered the University of Edinburgh early in 1719 during the middle of the colleges. Two University of Edinburgh records that place Bayes at the university in 1719 are both dated February 27, 1719. One of the records for Bayes on that day is the setting up of his library account, which amounted to £3. The other is a list, on which Bayes' name appears, of the students of Colin Drummond, the Professor of Logic and Metaphysics. On the record of library accounts, Bayes' name appears under the heading "supervenientes." These were students who had come from other universities or who had written examinations showing that they were qualified to attend a college partway through (Dalzel, 1862, Vol. II, page 184). Unless he came from a university such as Leiden, Bayes wrote the qualifying examinations and

began his studies about three or four months after lectures had started.

Another University of Edinburgh record shows that Bayes' entrance to the library was sponsored by James Gregory, the Professor of Mathematics. Unfortunately, there is no list of Gregory's students. However, Ward says in his letter dated May 10, 1720, after Bayes arrived in Edinburgh:

> The order which you follow in the rest of your studies I cannot but highly approve of. In occupying yourself simultaneously with both mathematics and logic you will more clearly and easily notice what and how much each of these excellent instruments contributes to the directing of thought and sensation.

That Bayes studied mathematics with James Gregory in the first year he arrived at Edinburgh is then a reasonable conclusion. Since it is likely that he arrived at Edinburgh mid-year, it is also likely that he had obtained some mathematical training before his arrival in order to write the examinations that would have allowed him to start late. Over 1720 and 1721 Bayes also attended lectures on history given by Professor Charles Mackie.

Gregory was part of a family of mathematicians. His uncle, an eminent mathematician also named James Gregory, had been Professor of Mathematics at Edinburgh in the 17th century. His brother David Gregory was another eminent mathematician. David Gregory had been Professor of Mathematics at Edinburgh until 1691 when he left his position and subsequently became Savilian Professor of Astronomy at Oxford. His brother James took over the professorship at Edinburgh in 1692 and remained in that position until 1725 when he retired due to age and ill health. Another brother, Charles Gregory, was Professor of Mathematics at the University of Aberdeen. The abilities of the James Gregory who taught Bayes were described by Grant (1884): "He seems to have been an able teacher, but did not otherwise add to the reputation of the Gregory family." James Gregory was succeeded at Edinburgh by Colin Maclaurin, who arrived there late in 1724.

The main reason that Bayes was at Edinburgh was to study divinity and to prepare for the ministry. Bayes entered Divinity Hall in 1720, probably in the fall after he arrived at the university; he appears on a list of students for 1720. Clearly, it was the family's intention that Thomas enter the ministry; it was his father who recommended him to the divinity school. By the time he left Divinity Hall he was licensed to preach but not ordained as a minister; presumably his ordination took place in London with the participation of his father. Thomas Bayes remained at Edinburgh until at least 1722. The latest recorded information on Bayes at Edinburgh is that, as part of his divinity training, he delivered analyses, or exegeses, of two different biblical passages. Both passages were sets of verses from the Gospel of Matthew. The earlier is dated January 14, 1721, and the later is dated January 20, 1722.

Bayes did not go to Edinburgh without family or friends. Attending the university at the same time were his friend Skinner Smith and his cousin Nathaniel Carpenter. They were both admitted to the Edinburgh University Library on January 27, 1719, a month before Bayes. Skinner Smith was another student of John Ward; he also received a letter from Ward in May of 1720. Like Bayes, Skinner Smith studied divinity at Edinburgh. Soon after leaving Edinburgh, Skinner Smith became the minister at the Old Dissenting Chapel in Cirencester. After holding this pastorate from 1726 to 1729, Skinner Smith moved to Abingdon where he remained until his death in 1748 (Murch, 1835; Summers, 1905). He was described as a "gentleman of great piety and learning, and a serious evangelical preacher." Not very much is known about Bayes' cousin Nathaniel Carpenter. He may have died young. When his father, also Nathaniel Carpenter, died in 1753, only the daughter Susannah was mentioned in the will.

After studies in Edinburgh, Thomas Bayes returned to London. There is, however, a six-year gap between the last Edinburgh records and the first London records. The first record from London dates from 1728. Thomas Bayes appears on a list of approved Presbyterian ministers in the London area submitted to the Body of Protestant Dissenting Ministers of the Three Denominations. This was a group or committee that included Presbyterians, Independents and Baptists. It met regularly to discuss problems that the denominations encountered. Joshua Bayes was often a member of this committee and sometimes chaired its meetings. In the 1728 list, Thomas Bayes was described as an approved minister, but unfixed in terms of a chapel or pastorate. He was also described as "residing at Mrs. Deacle's" rather than with any member of his family. She was probably the widow of a John Deacle, who died in 1723. This John Deacle was sufficiently wealthy to have a funeral sermon preached for him at the Presbyterian chapel in Crosby Square (Grosvenor, 1723).

Subsequently, Bayes became an assistant to his father at the Leather Lane chapel; he appeared this way on a 1732 list of approved Presbyterian ministers submitted to the Body of Protestant Dissenting Ministers (see also Wilson, 1808–1814 and the John Evans List). Bayes remained in London working with his father until perhaps late 1733 or early 1734, at which time he moved to Tunbridge Wells.

5. RELIGIOUS BELIEFS

Thomas Bayes was a Presbyterian minister and as such it is expected that he would hold religious views common to his fellow Presbyterians. However, among the Presbyterians of the 18th century there was a variety of religious positions that emerged early in the century. The extent of this variety may be set against a background of Christian orthodoxy. One of the tenets of orthodoxy was the doctrine of the Trinity. This doctrine holds that there are three natures to God: Father, Son and Holy Spirit. Of importance here is the relationship of God the Father and God the Son in the person of Jesus Christ. In the doctrine of the Trinity, Father and Son are co-equal and eternal. Departing from this path of Christian orthodoxy are beliefs that carry the labels of Arian, Socinian and Unitarian. Arians believed in a supreme God. God the Son, or Jesus, was a lesser God that was preexistent to his worldly birth. Socinians also believed in a supreme God, but held that Jesus was not preexistent and that he became a lesser God. Unitarians believed in one God only; the divinity of Jesus was denied. Arianism and Socinianism grew and flourished, though not without some tension, among Presbyterians throughout the 18th century. Unitarianism emerged only in the latter part of the century. See Wiles (1996) for a much fuller discussion.

Many Presbyterians in the 18th century strayed from orthodoxy on the doctrine of the Trinity. By the end of the century, many of the Unitarian chapels had evolved from, and taken over the buildings of, congregations that had previously been Presbyterian. This evolution, at least in the first half of the century, did not occur smoothly. One early episode in this evolution began 1719 in Exeter when two Nonconformist ministers were excluded from all Presbyterian pulpits in the city on suspicion of Arianism. A conference among the Three Denominations (Presbyterian, Congregationalist and Baptist) was held at the Salters' Hall meetinghouse in London to discuss the situation and to provide advice on how congregations should proceed in the future. The conference was deeply divided on how to handle the issue. Several prominent Nonconformist divines, including Joshua Bayes (James, 1867), abstained from taking part. The end result was that it was up to the individual congregations to determine the orthodoxy of their minister (Coomer, 1946). An example of the fallout over the Salters' Hall controversy is that the congregation at the Founders' Hall meetinghouse terminated the services of a Presbyterian minister, who for 19 years had given Sunday evening lectures at their chapel. His views in the controversy had differed with the views of their permanent pastor (Williams, 1922). What became apparent from the Salters' Hall controversy was that the power to appoint and dismiss a minister lay in the hands of the trustees of the meetinghouse and the pew-holders (those who paid rents on the pews) of the chapel.

Perhaps consistent with the fact that he declined to be involved in the Salters' Hall controversy over Arianism and how to handle it, Joshua Bayes was known as a moderate Calvinist who was tolerant of a variety of views (Wilson, 1814–1818). The theological views of Thomas Bayes are less certain though there is strong evidence of Arianism. The first hint comes from Timpson (1859), who stated that Bayes was not "evangelical in his doctrine." Fuller evidence comes by looking at who Bayes' friends (in a wide sense) were. A brief list that might substantiate the claim of Arianism includes James Foster, John Hoyle, Richard Price, William Whiston and perhaps Skinner Smith. I will deal with his friends in order of the increasing amount of evidence they provide toward Bayes' Arianism.

Skinner Smith and Bayes appear to have been friends while students at Edinburgh. Ward, in his letters to each of them, uses the Latin word *comes* (companion) to describe their relationship. As noted previously, Skinner Smith's first appointment was at the chapel in Cirencester. This was a Socinian chapel (Evans, 1897, page 661). However, there is nothing to connect Bayes to Skinner Smith after their university days together.

James Foster was a Nonconformist minister, originally from Exeter. The two Presbyterian ministers who were expelled from their pulpits in Exeter in 1719 were his friends. Foster probably also had Arian views (*Dictionary of National Biography*). His connection with Thomas Bayes is that Bayes, among many others, subscribed to the publication of Foster's last and greatest publication, *Discourses on all the Principal Branches of Natural Religion and Social Virtue* (Foster, 1749–1752). Without the subscriptions to this publication, Foster would have died penniless.

The case for John Hoyle is through Bayes' will. For Price, it is through the will and the fact that it was Price who presented Bayes' now famous paper to the Royal Society. With regard to wills, there was a tendency in the Bayes family to leave small amounts of money to Nonconformist ministers whom presumably the testator admired. Both Mary Bayes and Nathaniel Bayes each left £100 to Michael Pope. Four years prior to the death of their father Joshua, Michael Pope became the assistant at the Leather Lane Chapel. On Joshua Bayes' death, Pope became the pastor at Leather Lane. At her death, Mary Bayes lived in Stoke Newington, nowhere near the chapel where her father had been minister. In his will Thomas Bayes left £200 to be split between John Hoyle and Richard Price. Hoyle was the minister at Stoke Newington from 1748 to 1756. When Hoyle left Stoke Newington to take up a position in Norwich (Browne, 1877), Richard Price became the pastor at Stoke Newington. Both chapels eventually became Unitarian churches; both Hoyle and Price were known Arians (Evans, 1897).

The connection between Thomas Bayes and William Whiston (Figure 4) is that they had breakfast together at Bayes' home or lodgings in August of 1746. It was not the breakfast itself that is of importance, but the topic of conversation that Whiston reported (Whiston, 1749). The conversation centered on whether the Creed of Athanasius would be read in the Tunbridge Wells chapel associated with the Church of England. If so, Whiston was going to leave the service when this creed was read. Bayes informed Whiston that since the priest had not read that particular creed on the previously appointed day for it, which was Christmas day, it was unlikely to be read on the current day that was appointed. The problem with the Athanasian Creed for Arians was that it explicitly lays out in detail the doctrine of the trinity and ends with, as stated in the *Book of Common Prayer*, "This is the Catholic Faith, which except a man do faithfully and steadfastly believe, he cannot be saved." Whiston was an Arian (Wiles, 1996). He had begun his career as a Church of England vicar. He gave up his parish when he succeeded Newton in the Lucasian chair in Mathematics at Cambridge in 1703. In 1710, he was removed from his professorship because of his Arian views (*Dictionary of National Biography*). It is interesting to note that besides Bayes' probable Arian views, he knew what was happening in the local services of the Church of England.

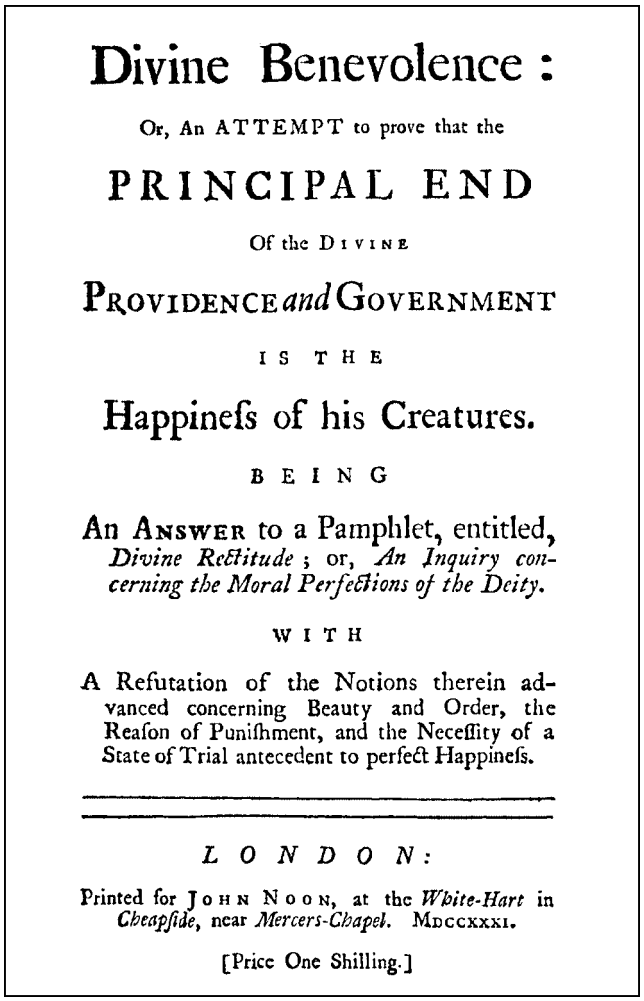
FIG. 4. *William Whiston* (*1667–1752*).

6. THEOLOGICAL WORK

Bayes' first publication was a theological work, entitled *Divine Benevolence* (Bayes, 1731, see Figure 5). Since no author appears on the title page of the book, or anywhere else, it is sometimes considered to be of doubtful authorship. For example, the *National Union Catalog* of the United States ascribes authorship to Joshua Bayes. However, Thomas Bayes was probably the author of this work. Bayes' friend, Richard Price refers to the book in his own work *A Review of the Principal Questions in Morals* (Price, 1948, page 248) and says that it was written by Thomas Bayes.

In *Divine Benevolence*, Bayes was trying to answer the question of the motivating source of God's actions in the world. The tract was written in response to a Church of England theologian, Dr. John Balguy, who claimed in his own writings, summarized by Doddridge (1822), that "God always does that which is right and fit, and that all his moral attributes, viz. justice, truth, faithfulness, mercy patience, &c. are but so many different modifications of rectitude." Bayes attributed the source of God's action in the world to God's goodness or benevolence. As noted by Pearson (1978), Bayes had a problem trying to explain this as the source of God's actions when there was pain and evil present in the world. To get around this problem, Bayes defined what he meant by "God's goodness" by first defining what he did not mean (Bayes, 1731, page 70):

> If we conceive of the goodness of God as an unbounded inclination to create happiness, and consequently suppose he has made the world as happy and as perfect as he possibly could, there are undoubtedly abundance of *phænomena*, the consistence of which with this supposition we cannot discern, and which we shall find some difficulty to perswade men are not compatible with it.



Divine Benevolence:

Or, An ATTEMPT to prove that the

PRINCIPAL END

Of the DIVINE

PROVIDENCE *and* GOVERNMENT

IS THE

Happineſs of his Creatures.

BEING

An ANSWER to a Pamphlet, entitled, *Divine Rectitude*; or, *An Inquiry concerning the Moral Perfections of the Deity.*

WITH

A Refutation of the Notions therein advanced concerning Beauty and Order, the Reaſon of Puniſhment, and the Neceſſity of a State of Trial antecedent to perfect Happineſs.

LONDON:

Printed for JOHN NOON, at the *White-Hart* in *Cheapſide*, near *Mercers-Chapel*. MDCCXXXI.

[Price One Shilling.]

FIG. 5.

In the next sentence he defined what he meant by divine goodness:

> But if we only conceive of the divine goodness a most kind affection towards his creatures, and as inclining him to confer upon that universe of creatures he has made the greatest happiness of which they are capable, still supposing that their original capacities were fixed by his will and pleasure, we shall find it much easier to satisfy our selves, that there is nothing in any appearances of providence contrary to the most perfect goodness of the divine nature.

Bayes' tract was followed by that of another Nonconformist minister, Henry Grove. He argued that the source of divine action was wisdom rather than rectitude or goodness.

By today's standards the argument seems almost irrelevant. At the time that it occurred, it drew considerable attention. In his own lectures on theology Philip Doddridge (1702–1751) referred to the controversy as "celebrated" (Doddridge, 1822). Richard Price, writing in the 1780s, lamented the fact that Bayes' tract was out of print (Price, 1948, page 248), and Toulmin (1814) described the tract as one that "excited attention." The controversy went out of fashion by the late 18th century. A footnote in Doddridge (1822), written by Andrew Kippis (1725–1795) says, "This controversy, though much celebrated in its time, is now nearly forgotten."

7. TUNBRIDGE WELLS

In Thomas Bayes' day, Tunbridge Wells was chiefly a tourist town. A very brief description of the town in the mid-18th century, as part of a larger description of the County of Kent, is from the November 1749 edition of *London Magazine* (page 492):

> Tunbridge [now Tonbridge], 7 miles S.E. of Sevenoak, has a market on Friday. The town of itself is but indifferent, and the streets ill paved; but what renders it famous, is the medicinal wells, about 5 miles from it, called Spelhurst [sic]-wells, but commonly Tunbridge wells, which occasion an annual resort of abundance of people of fashion, some for health, but more for diversion: And here many houses are built in a bottom between 2 hills, call'd Mount Sion and Mount Ephraim, with a handsome chapel of ease.

The chapel of ease refers to the local Church of England chapel that was built for the local residents, but which remained under the control of the parish church in nearby Speldhurst.

A wide variety of visitors or tourists arrived at Tunbridge Wells between the spring and fall every year. Writing in 1745, Elizabeth Montagu (1809, Vol. 3, pages 8 and 9), who came for both health and diversion, described the variety of people there. In terms of nationalities that year there were Hungarians, Italians, French, Portuguese, Irish and Scots. In terms of religious beliefs there were Jews and Roman Catholics as well as "quaint Puritans, and rigid Presbyterians." It is unlikely that she was ever in social contact with Bayes; she concluded her description of the variety of people at Tunbridge Wells with "I never saw a worse collection of human creatures in all my life. My comfort is, that as there are not many of them I ever saw before,

I flatter myself that there are few of them I shall ever see again." In 1749 she wrote more positively. To one friend she wrote (page 82), "the variety of persons and characters make Tunbridge an epitome of the world." To another (page 90) she wrote "Tunbridge seems the very parliament of the world, where every country and every rank has its representatives." She goes on to say to the same recipient, "For my part, I am diverted with the medley; the different characters are amusing, especially at the balls. . . ."

The wide variety of nationalities that arrived in Tunbridge Wells undoubtedly came through London. The metropolis was relatively close, about 36 miles (or 58 kilometers) away. Visitors could arrive either by private carriage or by the service available in London. From the 1730s to the 1760s, carriages came once a day to Tonbridge, not Tunbridge Wells, leaving London from the Bell Savage Inn in Ludgate Hill (*London Directories*). The service ran every day in the summer and on Mondays and Fridays only in the winter. Presumably travelers to Tunbridge Wells would change carriages at Tonbridge.

Among the many visitors to Tunbridge Wells was Philip Stanhope (1713–1786), 2nd Earl Stanhope (see Figure 6). A brief biography of him appears in *Public Characters*. Of relevance to the biography of Bayes are Stanhope's intellectual interests and family background. Stanhope's father had died when he was seven and he was put under the guardianship of his uncle Philip Dormer Stanhope (1694–1773), 4th Earl Chesterfield. Although the younger Stanhope was keenly interested in mathematics, Chesterfield thought the study of belles lettres was much more important and would not allow his ward to study mathematics. Once he came of age, Earl Stanhope took up mathematics with a great deal of enthusiasm and became an accomplished mathematician. Even before the age of 20, Stanhope's interests and abilities were described by a contemporary (Newman, 1969, page 105):

> [He] knows a great many things very well, but they are not such as young people generally have a relish for. He has read a good deal of Divinity, Metaphysicks, and Mathematicks. He is really pious, sober, chaste, and honest.

Stanhope had a London house in Duke Street. The family seat, however, was at Chevening near Sevenoak, about 12 or 13 miles (19–21 kilometers) from Tunbridge Wells. Chevening and Sevenoak were considered close enough to Tunbridge Wells that they both appear in some guides for the general area (e.g., Burr, 1766, page 226,).

Stanhope visited Tunbridge Wells in his early twenties, or perhaps sooner. There are two slightly different descriptions, by Elizabeth Montagu, of his visit in 1736. The descriptions are of the same visit, but from different editions of her letters. Climenson's (1906, page 18) version of Montagu's letter is:

> The person who was most taken notice of at Tunbridge as particular is a young gentleman your Grace may be perhaps acquainted with, I mean Lord Stanhope. He is always making mathematical scratches in his pocket-book, so that one half the people took him for a conjuror, and the other half for a fool.

An earlier edition (Montagu, 1809, Vol. 1, page 25) is slightly different in substance, but contains further information:

> The person most noticed for singularity at Tunbridge was Lord: he is always making mathematical scratches in his pocket-book, so that one half of the people took him for a conjuror. He is much admired and commended by his acquaintance, which are few in number. I think he had three at the Wells, and I believe he did not allow them above a sentence a piece in the whole day, the rest he left Lady—to say, who, I believe, doe not acquit herself ill of the office of spokeswoman. She seems to be very good natured, sensible, and of a more communicative temper than his lordship.

FIG. 6. *Philip, 2nd Earl Stanhope.*

It is uncertain when Bayes first met Stanhope. They definitely were on professional or social terms in Tunbridge Wells and they definitely discussed mathematical problems when they met. Interaction between the two, as recorded in Stanhope's papers, dates from the late 1740s.

For the people of fashion who were there for diversion, typical daily activities at Tunbridge Wells are described by Onely (1771), the rector of the parish church in Speldhurst:

> The morning is passed in an undress; in drinking the waters, in private or public breakfastings, which are sometimes given by one of the company, in attending prayers at the chapel [the chapel of ease], in social converse on the parade, at the coffee-house, in the public rooms, or bookseller's shop; in raffling for, cheapening and buying goods, at the milliners, turners, and other shops; billiards, cotillon dances, private concerts, cards, or some adventitious curiosity and novelty; a painter, a musician, a juggler, a fire-eater, or philosopher &c. After dinner, all go dressed to the parade again, and the rooms, to tea, in private parties, or in public—At night to a ball or assembly, and sometimes to a play. The ball nights are, *Tuesdays* and *Fridays*; and assemblies and cards every other night, except *Sundays*.

One of the "philosophers" who regularly came to town was William Whiston. After his ejection from Cambridge, described briefly in Section 5, he operated as a private chaplain and itinerant lecturer. In Tunbridge Wells, he initially preached at the chapel of ease (Barton, 1937, page 218). Because of his heretical views, he later hired rooms from which he gave lectures on millennial prophesies. He also made models of the tabernacle of Moses and the temple of Jerusalem, and lectured on these at Tunbridge Wells as well as other places. Bayes may have first met Whiston on one of his visits to Tunbridge Wells. They may also have met earlier in London prior to Bayes' move to Tunbridge Wells. Whiston lectured on astronomy and religious subjects in various London coffee houses early in the century (*Dictionary of National Biography*; Wilson, 1882).

As noted in *London Magazine*, the main part of the town was in a valley between two hills, Mount Sion and Mount Ephraim. The wealthy tourists tended to rent accommodation on the northern hill, Mount Ephraim. By the 1760s, the fashionable lodging places for the season had changed from Mount Ephraim to Mount Sion (Burr, 1766, pages 102–107). Of Mount Sion, Sprange (1780, page 7) wrote:

> A very good Presbyterian Meeting-House is situated about the middle [of Mount Sion]; and at the top of it [Mount Sion] a large grove of fine elms; which is frequently used by invalids and others both for walking or riding, when either the rays of the sun are two [sic] powerful, or the weather too precarious, to venture out to a greater distance.

The Presbyterian meetinghouse or chapel (see Figure 7) was built in 1720 and opened at the beginning of August of that year (Archer, 1720). The first minister of the chapel was John Archer. The license to have the chapel was obtained in April of 1721 (Kent Quarter Session Records). Among the trustees for the chapel was a man named John Jeffery. John Archer remained the minister of the Mount Sion Chapel until his death on September 23, 1733. Archer had been ill near the end of his life and his services were taken by some of his friends. Benjamin Mills, who preached Archer's funeral sermon a week after Archer's death, described how he came to preach the sermon (Mills, 1733, page 4):

> But it hath pleased God, in the Course of his sovereign Providence, to direct, that what I intended as an Assistance to my worthy Friend in his Inability for publick Service, should be changed into his Funeral Sermon.

Where the historical sources have commented, they have all been in agreement that Thomas Bayes suc-

FIG. 7. *Mount Sion meetinghouse in the 1990s.*

ceeded John Archer at the Mount Sion Chapel in Tunbridge Wells. There has, however, not been agreement on the time that Bayes arrived, with dates as early as 1731 being given; see Dale (1991a, page 395, ftn. 17) for a discussion. Bayes probably arrived in Tunbridge late in 1733 or early 1734, soon after the death of John Archer. None of the historical sources comments that there was a significant gap between Bayes' ministry and Archer's in Tunbridge Wells.

When Thomas Bayes came to Tunbridge Wells, being single he probably lodged with a Nonconformist family as he did with Mrs. Deacle in London. One likely location now bears the address 69 London Road (see Figure 8). The house, with ownership ascribed to John Jeffery, or Jeffry, appears on Bowra's 1738 map of Tunbridge Wells (Centre for Kentish Studies). This was the same John Jeffery who was a trustee of the Mount Sion Chapel. According to Roger Farthing of Tunbridge Wells, who searched deeds related to this house, it was a lodging house in Bayes' day and was owned by the Jeffery family from the late 17th century. John Jeffery's daughter Sarah subsequently owned the house. Bayes' tie to the family, and hence the house, is through his will. Bayes left a legacy to John's daughter Sarah. She received £500, a tidy sum of money in the 18th century, as well as "my watch by Ellicot and all my linen and wearing apparel and household stuff. . . ." Baptized on January 24, 1724, in Tunbridge, Sarah Jeffery was at least 20 years younger than Bayes. It is likely that the legacy to Sarah Jeffery was in gratitude for support that the family had given Bayes as trustees of the chapel. If Bayes did indeed hold Arian beliefs, then he needed the support of the trustees to retain his position as minister of the chapel. The legacy may also have recognized the support and help he would have received during illnesses that he had at times over the 25 years or more that he may have lodged with the Jefferys. Bayes, as he noted in a letter to Stanhope, was definitely ill, perhaps seriously, in 1755. All that is known further of Sarah Jeffery the legatee is that she married twice, both husbands also having the surname Jeffery.

FIG. 8. *69 London Road, Tunbridge Wells.*

Bayes also left money to two others named Jeffery—Richard and another Sarah Jeffery. These were the children of a Richard Jeffery and they shared a legacy of £100. They were also more than 20 years younger than Bayes. Sarah and Richard were baptized, also in Tunbridge, on October 17, 1726, and January 21, 1725, respectively. Although there are no parish records that I can find that would show it, the two fathers, John and Richard, were probably brothers. There is also a strong Nonconformist connection with the family of Richard Jeffery. Richard married a woman named Sarah, as did John Jeffery. Of interest here is that the maiden name of the Sarah who married Richard was Scoones; Thomas and John Scoones were also trustees of the Mount Sion Chapel when it opened in 1720. Also, when this Sarah Jeffery died in 1770, William Johnston, who was Bayes' successor at the Mount Sion Chapel, preached her funeral sermon (Johnston, 1771).

Although he was well respected as a minister (*Protestant Dissenter's Magazine*; Toulmin, 1814), Timpson (1859) says of Bayes that he was not a popular preacher. This would have been a distinct disadvantage to a Presbyterian minister. His main duties were connected to the Sunday services. A typical service had psalm singing, prayers, scripture reading and a sermon. Out of a service that would have lasted $1\frac{1}{2}$–2 hours, 1 hour was devoted to the sermon (Coomer, 1946). Many preachers spent considerable time preparing their sermons.

Though he remained in Tunbridge Wells until his death, Bayes gave up his ministry at the Mount Sion Chapel to William Johnston in 1752. If indeed Bayes was an Arian, the change in ministry also marks a distinct change in theology. Johnston was a pupil of Philip Doddridge (see Figure 9); Doddridge preached at the chapel when Johnston was installed as the minister. Doddridge was a leader of orthodoxy among the Presbyterians.

The presence of Doddridge and wills for various members of the Bayes family show that Thomas Bayes' family and some of his acquaintances did not share his probable Arianism. His father Joshua, uncle Nathaniel Carpenter and brother-in-law Thomas

FIG. 9. *Philip Doddridge* (*1702–1751*).

West all subscribed to Doddridge's *Family Expositor* (Doddridge, 1739–1756), a paraphrase of the New Testament with notes for further Bible study and reflection. As noted already, Thomas Bayes' brother Nathaniel and sister Mary left money to Michael Pope, who followed Joshua Bayes as the minister at Leather Lane. Pope was a popular minister, who was described as a liberal, but probably was orthodox (Wilson, 1808–1814, Vol. 4). John Ellicott, from whom Bayes bought his gold watch, and John Noon, who published both of Bayes' books, were both subscribers to the *Family Expositor*.

The change in ministry at the Mount Sion Chapel is also related to an event in the chapel that began 3 years before. By the mid-1740s there was a desire for an Independent chapel in Tunbridge Wells. It had grown out of some small house meetings for scripture reading and prayer. In 1749 some Independents, according to Timpson (1859),

> engaged the Presbyterian chapel, from the Rev. Mr. Bayes, its minister. They enjoyed the gospel preached by ministers sent from London for nearly a year, until Easter Sunday in 1750, when Mr. Bayes resumed his pulpit, disliking the doctrine of the Independents and they again attended at the Established church, for the sake of the Lord's Supper.

The next year, the principal players from the Independents' side, who are described in Timpson (1859), obtained a license to have a dissenting meetinghouse associated with one of their own homes. In 1752 another license was obtained for a new building to serve as an Independent chapel (Kent Quarter Session Records).

Strange (1949) interpreted the events surrounding the Independents' use of Mount Sion Chapel as a desire on the part of Bayes to retire from the ministry; Holland (1962) adopted this interpretation. In view of William Johnston's orthodoxy following on Bayes' Arianism, the two events may also be interpreted as mounting tension within the Presbyterians of Tunbridge Wells over Bayes' probable heterodoxy. Since he was independently wealthy and did not need the financial support of his congregation at the chapel, Bayes bowed out of the ministry. As will be seen in Section 10, illness may also have been a factor.

When Joshua Bayes died, he left his son Thomas not only a sizable fortune, but also his library. At Thomas Bayes' death, his library went to William Johnston. Timpson (1859) states, "He bequeathed his valuable library to his successor, the Rev. William Johnson [sic], M.A., who became minister of the chapel in 1752." There is no mention of this bequest in Bayes' will. It is likely that it was a gift made by Bayes' executors, his brother Nathaniel and his nephew Joshua Cotton. Following on the Arian versus orthodox divide within the family, Nathaniel Bayes may have wanted at least his father's theological books to go to someone with orthodox religious views within Presbyterianism.

8. ELECTION TO THE ROYAL SOCIETY

Before being put up for election to fellowship in the Royal Society, it was normal to attend a meeting of the Society under the sponsorship of another fellow. In this way the candidate could be introduced to those who would vote on the nomination. Such was the case with Bayes. John Belchier, an eminent surgeon at Guy's Hospital in London, brought Bayes to his first meeting on March 25, 1742. Two weeks later on April 8, Bayes' nomination certificate was signed by Philip Stanhope, 2nd Earl Stanhope, followed by Martin Folkes, Sir James Burrow, Cromwell Mortimer and John Eames. Interestingly, the minutes of the meeting (Royal Society, *Journal Books*) show that Eames had not signed the certificate, but the surviving certificate has his signature on it. It is likely that Eames signed the certificate at the end of the meeting after the minutes had been recorded. This would indicate that Eames was not the primary sponsor for Bayes' fellowship, but knew of him or his work. The certificate reads:

> The Revd Thomas Bays of Tunbridge Wells, Desiring the honour of being Elected into this Society, we propose and recommend him as a Gentleman of known merit, well skilled in Geometry and all parts of Mathematical and Philosophical Learning, and

> every way qualified to be a valuable member of the same.

It was posted so that fellows could politic over the upcoming election. The election took place on November 4 and Bayes was duly elected. He was admitted as a Fellow a week later. On his admission he paid the normal admission fee and then an additional amount of 20 guineas as a lifetime payment in lieu of regular dues.

Pearson (1978) has been the one who has speculated the most about this election. Unfortunately Pearson's discussion is tainted with the misapprehension of what constituted the mathematical work that Bayes had published anonymously. Most biographers of Bayes attribute the nomination and election to the book published anonymously in 1736 (Bayes, 1736). Pearson was unsure of the publication and mentioned two other books, one published in 1741 and the other in 1751. With regard to the certificate and the signatures that appear on it, Pearson (1978) commented, "Now these appear to be the most extraordinary set of names to be attached to the certificate of the minister of a non-conformist chapel in Tunbridge Wells!" Pearson (1978) wondered how a scientific nonentity, who was not part of the political, social or ecclesiastical establishment, could be put forward by some of the establishment itself. At the time of the election, Folkes was the President of the Society and Mortimer was the Secretary. Over 25 years later, Burrow became President of the Society. To try to address this issue, Pearson examined some of the known religious beliefs of the Fellows to ascertain what the prevalence was of those with a Nonconformist religious persuasion in the Society. He came up with a very short list and took the discussion no further. However, Pearson concluded quite rightly about the names on the certificate: "With such names on his certificate Bayes was certain of election." It is then a useful exercise to examine Bayes' sponsors more closely since there are some close and some tenuous connections.

At the top of the list is Philip Stanhope (1713–1786), 2nd Earl Stanhope; see *Public Characters* for a biography. As noted in Section 7, Stanhope was keenly interested in mathematics, even after his uncle had denied him the opportunity to study the subject in his youth. It was probably Stanhope who read Bayes' defense of Newton's calculus or doctrine of fluxions (Bayes, 1736) and decided that Bayes would be a good candidate for fellowship. Stanhope was both a promoter of mathematics and of mathematicians. For example, on the death of Robert Smith (1687–1768), a mathematics professor at the University of Glasgow, Stanhope paid for the publication of Smith's posthumous works and sent copies of these works to every learned society in Europe and every prominent mathematician that he knew. Furthermore, the Centre for Kentish studies holds Stanhope's mathematical papers, which contain his correspondence with several leading mathematicians of the day. Stanhope was also a patron of known Nonconformists. Joseph Priestley (1733–1804), who held Arian views from about 1750 to 1765 until he became a Unitarian (Wiles, 1996, page 148), dedicated the third volume of one his books related to experiments on air to Stanhope. It is then not surprising that Stanhope would not only support, but also initiate, Bayes' nomination to the Royal Society.

Martin Folkes (1690–1754), the next on the list, is known in history as an antiquary. In his earlier years, as a student at Cambridge, Folkes excelled in mathematics and philosophy. His first publication was in astronomy. Folkes became a friend of Newton, who nominated Folkes for the vice presidency of the Society. He was also a friend of Robert Simson, Plumian Professor of Mathematics at Cambridge. When Simson was writing his book *A Complete System of Optics*, Folkes provided Simson with several comments on his work so that Folkes' contributions were acknowledged in the preface of the published book. Stanhope was also aware of Folkes' mathematical abilities and corresponded with him on mathematical questions.

The connections of James Burrow (1701–1782) and Cromwell Mortimer (d. 1752) to mathematics or to nonconformity are both tenuous. Burrow was a lawyer, who had no apparent connection to either. Mortimer's connections are only slightly better than Burrow's. He was a physician, taking his doctorate in medicine in 1724 at the University of Leiden, one of the locations of study for sons of Nonconformist families.

John Eames (d. 1744), last on the list, shared some of Bayes' interests, both theological and scientific. Eames had trained for the ministry as a Nonconformist, but because of a speech defect gave it up to teach classics and science, which included mathematics, in one of the dissenting academies. This was the Fund Academy in Tenter Alley in London founded by the Congregational Fund Board in 1695. Eames also taught theology. He was appointed assistant tutor in theology at the Academy in 1712 and was made tutor in 1734. Eames was active in the Royal Society and was both friend and colleague to Isaac Newton. He also participated in experiments with John Ellicott, the watchmaker (Royal

Society, *Journal Books*). In his youth Bayes may have met Eames when he was studying at Ward's school, which was also in Tenter Alley.

The most likely background for Bayes' election is that Stanhope, hearing of Bayes' interest in mathematics, met Bayes in the 1730s in Tunbridge Wells, which was near his family estate. Stanhope obtained a copy of *The Doctrine of Fluxions* (Bayes, 1736), either prior to their first meeting, which perhaps prompted it, or after their meeting. Thus began an association that led to Bayes' election to the Royal Society. *The Doctrine of Fluxions* (Bayes, 1736), as a publication on its own, remains a problem in accounting for Bayes' election. The nomination papers, in part, read, "well skilled in Geometry and all parts of Mathematical and Philosophical Learning." *The Doctrine of Fluxions* (Bayes, 1736) is a work on the foundations of fluxions or differential calculus and is not a work of geometry. I (Bellhouse, 2002) argued that Bayes' "test work" for entry into the Royal Society was an unpublished paper on the topic of trinomial divisors, a paper that uses geometrical arguments to obtain its results. The undated paper, copied in Stanhope's hand but attributed to Bayes, is among the Stanhope papers in the Centre for Kentish Studies. The problem attacked in the paper is to find the n factors, in terms of second degree polynomials, of the trinomial expression $x^{2n} - 2\cos(\theta)x^n + 1$ for any angle $0 < \theta < 2\pi$. Since Maclaurin (1742), also using geometrical arguments, solved the problem, I (Bellhouse, 2002) argued that Bayes' paper predates 1742 and the election. *The Doctrine of Fluxions* (Bayes, 1736) becomes a catalyst for election to the Royal Society and the paper on trinomial divisors is the deciding factor. Stanhope may have sent the paper to Folkes for his perusal and approval. Initially they were responsible for bringing forward Bayes' nomination. They then pressed Burrow and Mortimer into service to assure the success of the nomination. At the meeting of April 8 when the nomination was made, Eames, knowing Bayes, joined in.

9. SCIENTIFIC WORK AND INTERESTS

9.1 Interest in Newton's Doctrine of Fluxions

Thomas Bayes was a strong Newtonian in his scientific outlook. The first indication is that as a subscriber he supported the publication of Pemberton's (1728) *A View of Sir Isaac Newton's Philosophy*. Newton's philosophy refers to natural philosophy, the general phrase used for science in the 18th century, most particularly for physics. Henry Pemberton (1694–1771) was

FIG. 10. *George Berkeley* (*1685–1753*).

one of the great popularizers of Newton's work. He had been employed by Newton to oversee the production of the third edition of Newton's *Principia* (*Dictionary of National Biography*). Bayes may have first come across Newtonian natural philosophy as a student at Edinburgh. David Gregory, who had been Professor of Mathematics at Edinburgh until 1691, was the first to lecture publicly on Newtonian philosophy at Edinburgh. His brother James, who succeeded him as Professor of Mathematics and who taught mathematics to Bayes, would likely have lectured on Newtonian philosophy as well, especially considering that his brother's lecture notes remained at Edinburgh (*Dictionary of National Biography*).

In 1734, George Berkeley (see Figure 10), the eminent philosopher, published an attack on the logical foundations of Newton's doctrine of fluxions or differential calculus (Berkeley, 1734). It was described by Cajori (1919) as "the most spectacular event of the century in the history of British mathematics." Berkeley's argument was based on the derivative of x^n. The ratio of the increments of x^n to x is given by

$$\frac{(x+h)^n - x^n}{(x+h) - x} \doteq nx^{n-1} + h\frac{n(n-1)}{2}x^{n-2},$$

upon expanding the term $(x+h)^n$ and dropping higher powers of h. The limit as h goes to 0 yields the derivative nx^{n-1}. The jargon of the time was to let h vanish. Berkeley's concern was that if the increment vanished, then one did not really have an increment in the first place. The expansion of $(x+h)^n$ is based on having an increment so that there is a contradiction and hence the whole doctrine falls apart. Smith (1980) explained the problem simply:

> The attack rested mainly on the assertion that Newton has assumed a quantity to be

> simultaneously zero and non-zero, and that no valid deductions could be based on such contradictory assumptions.

According to Smith, the real problem is that Newton had not expressed his ideas on fluxions rigorously enough. Several mathematicians rose to Newton's defense including Thomas Bayes. What Bayes did (Bayes, 1736) was to provide the necessary rigor. Smith (1980), who has provided a detailed analysis of Bayes' treatise, sums up Bayes' work:

> Bayes began with a careful discussion of the meaning of fluxions and prime and ultimate ratios. He proceeded to develop the properties of prime and ultimate ratios in a way not unlike Cauchy's treatment of limits, using these results to prove some basic theorems concerning the calculation of fluxions.

On the other hand, Jesseph (1993) was much more negative in his assessment of Bayes' work. While acknowledging that Bayes' work foreshadowed later rigorous work on the calculus, Jesseph found Bayes' discussion of ultimate ratios obscure and felt that Bayes failed to address Berkeley's main criticisms.

9.2 Work on Infinite Series

Thomas Bayes' early work appears to have been related mainly to infinite series, which was one of the paths followed by British mathematicians in the 18th century (Guicciardini, 1989). His early work appears to be motivated by the desire to obtain a correct derivation of Stirling's approximation to factorials.

Bayes' earliest known mathematical work after his anonymous publication (Bayes, 1736) is a result related to infinite series and numerical analysis. It appears as a note written by Stanhope on a scrap of paper that is among Stanhope's surviving papers in the Centre for Kentish Studies:

> Theorem mentioned to me at Tunbridge Wells by M^{r} Bayes Aug. 12. 1747.
>
> $$\dot{y} = y - \tfrac{1}{2}\,\underset{\cdot}{y} + \tfrac{1}{3}\,\underset{\cdot\cdot}{y} - \tfrac{1}{4}\,\underset{\cdot\cdot\cdot}{y} + \tfrac{1}{5}\,\underset{\cdot\cdot\cdot\cdot}{y} - \tfrac{1}{6}\,\underset{\cdot\cdot\cdot\cdot\cdot}{y} + \&c.$$

The dot over the y denotes the fluxion or differential dy/dt and the number of dots under the y denotes the order of differencing in terms of Newton's forward differences, for example, $\underset{\cdot}{y} = \Delta y = y(t+1) - y(t)$. This result and a related one,

$$y = \dot{y} + \tfrac{1}{2}\,\underset{\cdot}{\dot{y}} + \tfrac{1}{2.3}\,\underset{\cdot\cdot}{\dot{y}} + \tfrac{1}{2.3.4}\,\underset{\cdot\cdot\cdot}{\dot{y}} + \&c,$$

which together provide the relationship between derivatives and finite differences, are the very first results that appear in Bayes' undated notebook. The notebook is preserved in the muniment room of The Equitable Life Assurance Society. The second result is basically a Taylor series expansion of $y(t+h)$ about t, where $h = 1$. The methodology appears in Taylor (1715) and undoubtedly would have been known to Bayes. It also explains why Stanhope would have referred to the first result as Bayes' theorem and not the second. Bayes did not provide a proof of either of these results, but instead used them in the notebook to provide a derivation of "what is essentially the Euler–Maclaurin sum formula" (Dale, 1991b). Bayes never published the theorems relating differences and derivatives. The first publication that I can find related to these results is attributed to Lagrange in 1772 and again in 1792 (Lagrange, 1869–1870); see also Goldstine (1977, pages 164 and 165) for a discussion. Lagrange's result is more general, giving the left-hand side of either equation as a general order of derivative or difference. Bayes claimed to have obtained the general result, stating in his notebook that "y^{e} relation between $\ddot{x}$ & $\underset{\cdot\cdot}{x}$ & so on may be found." A detailed discussion of the contents of Bayes' work as it appears in the Stanhope papers is given in Bellhouse (2002). Of consequence here is that it may be safely assumed that Bayes started this notebook in 1747.

As shown by Dale (1991b) the infinite series relating derivatives and finite differences are connected to Bayes (1763b) in which Bayes showed that a particular series is divergent. The series had been used to obtain an approximation to $\log(z!)$ or equivalently Stirling's approximation to factorials. The divergent series appears in two places in the Stanhope collection among a set of papers labeled "Mathematical papers of Mr. Bayes' communicated Sept^{r}. 1st. 1747." One of the manuscripts opens with the following statement:

> It has been asserted by several eminent Mathematicians that the sum of the Logarithms of the numbers 1. 2. 3. 4. 5 &c to z is equal to $\frac{1}{2}\log, c + \overline{z + \frac{1}{2}} \times \log, z$ lessened by the series $z - \frac{1}{12z} + \frac{1}{360z^3} - \frac{1}{1260z^5} + \frac{1}{1680z^7} - \frac{1}{1188z^9} + \&c$ if c denote the circumference of a circle whose radius is unity.

In modern notation,

$$\tfrac{1}{2}\log, c + \overline{z + \tfrac{1}{2}} \times \log, z \equiv \tfrac{1}{2}\ln(c) + \left(z + \tfrac{1}{2}\right)\ln(z).$$

This quotation is the second paragraph verbatim of the letter from Bayes to Canton on the divergent infinite series that was published posthumously (Bayes,

1763b). Another manuscript within the same group is a derivation by Bayes of an approximation to $z!$, given by $\sqrt{2\pi z}z^z e^{-z}$, which is Maclaurin's (1742) form of the Stirling approximation. Maclaurin had obtained his version of the approximation using the divergent series, while Bayes' approach to the approximation does not rely on the divergent series. On examining Bayes' notebook in which there are results related to the divergent series as well as notes, or partial transcriptions (see Dale, 1991b), from articles 827, 839, 842 and 847 of Maclaurin (1742), it is apparent that Bayes was motivated to look at this divergent series after seeing Maclaurin's incorrect derivation of the approximation to $z!$. It may also be noted that Bayes had obtained the posthumously published result of infinite series at least 15 years prior to its publication. He never published the correct derivation of $z! \sim \sqrt{2\pi z}z^z e^{-z}$.

9.3 Interest in Probability

There has been much speculation as to when Bayes first became interested in probability theory or how he learned probability. For example, Barnard (1958) suggested that Bayes could have learned mathematics, and implicitly probability, from de Moivre (see Figure 11). Stigler (1986) more reasonably suggested that Bayes became interested in probability after reviewing a publication of Thomas Simpson (see Figure 12). Essentially, what Simpson (1755) had proved was a special case of the law of large numbers: the mean of a set of observations is a better estimate of a location parameter than a single observation. In a letter to John Canton, Bayes pointed out what in modern parlance is that this may not be true in the presence of measurement bias. In Bayes' words,

> Now that the errors arising from the imperfection of the instruments & the organs of sense shou'd be reduced to nothing or next to nothing only by multiplying the number of observations seems to me extremely incredible. On the contrary the more observations you make with an imperfect instrument the more certain it seems to be that the error in your conclusion will be proportional to the imperfection of the instrument made use of. For were it otherwise there would be little or no advantage in making your observations with a very accurate instrument rather than with a more ordinary one, in those cases where the observation cou'd be very often repeated: & yet this I think is what no one will pretend to say.

FIG. 11. *Abraham de Moivre* (*1667–1754*).

FIG. 12. *Thomas Simpson* (*1710–1761*).

Stigler (1986, pages 92–95) gave a full description of Simpson's result as well as the effect that Bayes' comments had on a later publication by Simpson.

Stigler's view of the origin of Bayes' interest in probability is supported by two letters involving Stanhope. The first is a letter from Stanhope to Martin Folkes, which is preserved in the Royal Society Library. Although undated, the letter is in a collection containing correspondence mainly from the late 1730s to the early 1740s. In this letter, Stanhope wrote to Folkes posing a problem in probability:

> It is disputed at White's, whether it be an equal wager to lay that the Dealer at whist will have four Trumps. Some think it disadvantageous to lay on the Dealer's side, because he has but 12 cards left wherein to find Trumps, when all the others have 13 apiece for them. Others say, but I don't understand how they can prove it that the advantage to lay on the Dealers side amounts to 25 per cent. If you are at leisure, I should be glad to know your opinion, but if otherwise employ'd perhaps Mr. Daval at your request might take the trouble to consider it.

White's Coffee House (later White's Club) had gained a reputation for betting and gambling as early as 1739 (Lillywhite, 1963, page 642). What is significant here is that Stanhope asked Folkes for help, not Bayes, whom he probably knew at the time the letter was written. Furthermore, failing Folkes, Stanhope wanted Peter Daval and not Bayes to help out. Daval was a barrister as well as a Fellow of the Royal Society and an able mathematician (Chalmers, 1812–1817). A second letter is from Patrick Murdoch to Stanhope and is dated March 18, 1755. Murdoch was a clergyman in the Church of England; he had studied mathematics under Maclaurin in Edinburgh (*Dictionary of National Biography*). The letter, which is part of the Stanhope papers, points to wider group of mathematicians who were unaware of any interests in probability that Bayes might have had. After acknowledging comments made by Bayes on a paper that Murdoch had written, Murdoch made some proposals regarding the publication of de Moivre's (1756) *Doctrine of Chances*:

> The edition which Mr de Moivre desired me to make of his Chances is now almost printed; and a few things, taken from other parts of his works, are to be subjoined in an Appendix. To which Mr Stevens and some other Gentlemen, propose to add some things relating to the same subject, but without naming any author: and he thought, if your Lordship was pleased to communicate any thing of yours, it would be a favour done the publick. Mr Scott likewise tells me, there are in your Lordship's hands two Copy Books containing some propositions in Chances, which de Moivre allowed him to copy. If your Lordship would be pleased to transmit these (to Millar's) with your judgement of them, it might be a great advantage to the Edition.

There is a request to Stanhope to submit his work in probability, but no mention of Bayes, who was acknowledged at the beginning of the letter for his comments on Murdoch's mathematical work. An interesting sidelight is that, on examining the publication by de Moivre (1756), there seem to be very minor additions in an appendix to the publication beyond what appears to be de Moivre's own work.

9.4 Other Scientific Interests

Bayes had several areas of scientific interest beyond infinite series and probability. Dale (n.d.), in a discussion and partial transcription of Bayes' notebook, pointed to four broad areas: mathematics, natural philosophy, celestial mechanics and a miscellaneous category. Within mathematics, the notebook contains additional material related to trigonometry, geometry, solutions of equations and differentials. The material on natural philosophy relates to electricity, weights of bodies, optics and harmony in music. Related to material in the notebook on electricity is a letter to John Canton, preserved in the Royal Society Library, written partially in longhand and partially in shorthand. The letter contains notes on a work on electricity by Hoadly and Wilson (1756). The part written in longhand contains critical comments on the work. Home (1974) provided a discussion of Bayes' electrical work. In the miscellaneous category in Bayes' notebook are extracts from works of others on a variety of topics ranging from the measurement of the pyramids to Pascal's *Lettres provinciales*.

Bayes' general scientific interests fit in with an unsubstantiated story reported in Phippen (1840):

> During the life of Mr. Bayes, an occurrence took place which is worthy of record. Three natives of the East Indies, persons of rank and distinction, came to England for the purpose of obtaining instruction in English literature. Amongst other places, they visited Tunbridge Wells, and were introduced to Mr. Bayes, who felt great pleasure in furnishing them with much useful and valuable information. In the course of his instructions, he endeavoured to explain to them the severity of our winters, the falls of snow, and the intensity of the frosts, which they did not appear to comprehend. To illustrate in part what he had stated, Mr. Bayes procured a piece of ice from an ice-house, and shewed them into what a solid mass water could be condensed by the frost—adding that such was the intense cold of some winters, that carriages might pass over ponds and even rivers of water thus frozen, without danger. To substantiate his assertion, he melted a piece of ice by the fire, proving that is was only water congealed. 'No,' said the eldest of them, 'It is the work of Art!—we cannot believe it to be anything else, but we will write it down, and name it when we get home.

While the story is also in line with the variety of nationalities that came to Tunbridge Wells, as described by Elizabeth Montagu in Section 7, it may very well be apocryphal. Dale (1991a, page 13) noted that similar stories of people from warm climates encountering ice are to be found in the works of John Locke and David Hume. Locke's *An Essay Concerning Human Understanding*, which went through five editions between 1690 and 1706 (Locke, 1975), contains a story of an interaction between a Dutch ambassador and the King of Siam in which the ambassador describes the ability to walk on ice during cold weather (Book IV, Chapter 15, Section 5). Later in 1750, Hume turned the King of Siam into an Indian prince and focused on the effect that encountering ice for the first time might have on the prince. The story given by Hume and a discussion of various versions of the story by others is found in Hume (2000, pages 86 and 172).

9.5 Bayes as Referee and Critic

Within five years of his election to the Royal Society in 1742, Bayes was known, at least to Stanhope, for his work in infinite series. His reputation grew beyond Stanhope. Bayes' letter to John Canton on the paper by Thomas Simpson, probably written in 1755, shows that Bayes was initially looking at the correctness of Simpson's mathematics, for he says, "Hence therefore as I see no mistakes in Mr. Simpsons calculations I will venture to say that there is one in the hypothesis upon which he proceeds." It appears that Bayes played the role of critic or commentator for a network of mathematicians that initially centered on Stanhope and/or John Canton.

Just as Bayes was providing comments to Canton on Simpson's work, he was also commenting to Stanhope on a paper by Patrick Murdoch. Initially, it appears that Stanhope had sent Bayes a copy of Murdoch's paper to look at. Then, after Stanhope had received Bayes' comments, he forwarded them to Murdoch. From that exchange some correspondence ensued. Neither Murdoch's paper nor Bayes' actual comments are in the Stanhope papers. The sticking point in the discussion over the paper was an interpretation of the use of Propositions 44 and 45 in Newton's *Principia* (Newton, 1969). Proposition 44 deals with the difference of the forces necessary to have two bodies in orbit move equally when one body is revolving and the other is not, while the next proposition is concerned with the motion of the apsides of orbits that are close to circles. Here are excerpts in an exchange of correspondence involving Stanhope, Murdoch and Bayes. The first is from Murdoch to Stanhope dated March 18, 1755:

> I am ashamed not to have sooner acknowledged your Lordship's goodness in communicating to me Mr Bayes' paper, which I received from Dr Pringle. But during my short stay in town I was much hurried; and since my return to the Country have had as little time to think of those subjects. I have now returned it inclosed, with my answer on the blank page: which I wish our Lordship and Mr Bayes may find satisfactory.

After the opening sentence, Bayes wrote on April 25:

> I am much obliged to your Lordship for the communication of them [Murdoch's answers] as well as the kind promise to transmit to him any thing farther I might have to say on the subject. It wou'd be a greater pleasure to go on, where there hopes of soon coming to an agreement, or of seeing difficulty removed without giving too much trouble to Mr. M. But for fear this shou'd not be, I don't at present think of entering upon any new point & that which is now upon the carpet will I hope be brought pretty near to a conclusion by the adjoining paper.

Murdoch replied on May 11:

> I received the honour of your Lordship's letter of April 27th, with Mr Bayes' paper: to which I do not think any answer necessary, seeing we seem to be agreed on that point. That ever Mr B. misunderstood me was certainly my own fault, in not explaining more particularly the sense in which I took Sr Isaac's Corollary. . . . His 44 Proposition, your Lordship knows is general and holds good in finite cases: but to apply it to the uses he had in view, needed the masterly artifice he uses in the 45th, and its Corollaries: which consists in reducing every disturbing force that is expressible by the distance, to the denomination $1/43$; and this could not be done but in the case of near coincidence with a Circle, when the higher Terms of the series vanish. So that the Canon which he deduces in this manner is really the result of a fluxionary Equation; and consequently has a form that does not suit finite cases. When Mr B. considers it in this light he can have no further difficulty: and instead of taking

> any thing amiss on this occasion, I reckon myself very much indebted to him: as the kindest thing one man can do by another is correcting his mistakes. I shall take particular care that the copy books be returned to your Lordship.

Bayes' notebook gives some additional evidence for the existence of this network. The notebook shows that Bayes was probably corresponding with Robert Smith of the University of Glasgow some time after 1749. Smith's connection to Stanhope has already been noted in Section 8.

Despite the fact that his only publication in mathematics during his lifetime was anonymous and relatively early in his career, Bayes was well regarded by the mathematical community of his day. One indication is the wording of Bayes' certificate for election to the Royal Society in which he is described as "well skilled in Geometry and all parts of Mathematical and Philosophical Learning." This is echoed by some of Bayes' friends. Price (1948) described Bayes as "one of the most ingenious men I ever knew" and Whiston (1749) noted that Bayes was "a very good mathematician."

9.6 Richard Price and Bayes Theorem

Bayes' fame rests on a result in probability that was published posthumously (Bayes, 1763a). It was Bayes' friend Richard Price (Figure 13) who communicated the paper through John Canton to the Royal Society two years after Bayes' death in 1761. One reason for the delay in publication was Price's personal circumstances at the time. In 1758 Price was made the minister at a dissenting chapel in Newington Green. The same year he also moved to that area of London to be near his congregation. There he tried to lead a quiet life, dividing his time between study and his congregation. Price's nephew, William Morgan, described Price's social interactions at this time as (Morgan, 1815, page 20):

> Excepting Dr. [Benjamin] Franklin, Mr. [John] Canton, and two or three other philosophical friends, his acquaintance at this period was chiefly confined to members of his own congregation.

It is unlikely that Bayes was one of the friends that Morgan mentions, since Bayes in his will refers to Price as, "now I suppose preacher at Newington Green." A further possible cause for the delay in publication is that Price's wife became ill in 1762 and suffered from this ailment for the rest of her life.

FIG. 13. *Richard Price* (*1723–1791*).

A third cause for the delay is that Price was dissatisfied with Bayes' solution. A description of Price working on Bayes' paper appears in Morgan (1815, pages 24–26):

> On the death of his friend Mr. Bayes of Tunbridge Wells in the year 1761, he was requested by relatives of that truly ingenious man, to examine the papers which he had written on different subjects, and which his own modesty would never suffice him to make public. Among these Mr. Price found an imperfect solution of one of the most difficult problems in the doctrine of chances, for 'determining from the numbers of times an unknown event has happened and failed, the chance that the probability of its happening in a single trial lies somewhere between any two degrees of probability that can be named.' The important purposes to which this problem might be applied, induced him to undertake the task of completing Mr. Bayes' solution; but at this period of his life, conceiving his duty to require that he should be very sparing of the time which he had allotted to any other studies than those immediately connected with his profession as a dissenting minister, he proceeded very slowly with the investigation, and did not finish it till after two years; when it was presented by Mr. Canton to the Royal Society, and published in their Transactions in 1763.—Having sent a copy of his paper to Dr. Franklin, who was then in America,

> he had the satisfaction of witnessing its insertion the following year in the American Philosophical Transactions.—But not withstanding the pains he had taken with the solution of this problem, Mr. Price still found reason to be dissatisfied with it, and in consequence added a supplement to his former paper; which being in like manner presented by Mr. Canton to the Royal Society, was published in the Philosophical Transactions in the year 1764. In a note to his Dissertation on Miracles, he was availed himself of this problem to confute an argument of Mr. Hume against the evidence of testimony when compared with regard due to experience; and it is certain that is might be applied to other subjects no less interesting and important.

This quotation opens up a number of questions. Did Bayes' paper also appear in an American publication? Is there a copy of the manuscript among Benjamin Franklin's papers? What motivated Price to work on this particular paper from among the papers that became available to him? With what part of Bayes' original paper was Price dissatisfied so that the solution was "imperfect"?

The first two questions are relatively easy to answer. No in both cases. The first negative is based on checking indexes to American periodicals from the 18th century (Anonymous, 1986) as well as other reference books on American periodical literature at the time, especially in Philadelphia where Franklin lived. The second negative is based on Franklin's surviving correspondence and papers for that period (Franklin, 1959). The editors of Price's correspondence (Price, 1983, page 56) noted that Morgan "is not very reliable in his dating" of some events in Price's life. It may well be that the paper that Morgan refers to is Price's paper on reversionary annuities (Price, 1769), which was sent to Franklin.

What probably motivated Price to work on Bayes' manuscript were the theological implications that Price perceived in the result. At this time in his life, Price was deeply immersed in theological and philosophical study. Price notes in Bayes (1763a) that Bayes had written an introduction to the paper; but Price did not include Bayes' introduction and instead supplied his own. In other manuscripts of Bayes that I have seen (Bellhouse, 2002), Bayes typically gives no motivation for the mathematical results that he presents. The same may be true for his essay on probability. Price only says of Bayes that:

> . . . his design at first in thinking on the subject of it was, to find out a method by which we might judge concerning the probability that an event has to happen, in given circumstances, upon supposition that we know nothing concerning it but that, under the same circumstances, it has happened a certain number of times, and failed a certain other number of times. He adds, that he soon perceived that it would not be difficult to do this. . . .

Later in the introduction to Bayes (1763a), Price states that:

> Every judicious person will be sensible that the problem now mentioned is by no means merely a curious speculation in the doctrine of chances, but necessary to be solved in order to a sure foundation for all our reasonings concerning past facts. . . .

Further on in the paper, after discussing de Moivre's work, Price states:

> The purpose I mean is, to shew what reason we have for believing that there are in the constitution of things fixt laws according to which events happen, and that, therefore, the frame of the world must be the effect of wisdom and power of an intelligent cause; and thus to confirm the argument taken from final causes for the existence of the Deity.

What motivated Price to work on this paper was that to him the result provided a proof of the existence of God. Price came back to this theme in his theological work *Four Dissertations* (Price, 1767), which is mentioned by Morgan in the context of refuting Hume. A discussion of Price's argument was given by Thomas (1977, pages 133 and 134).

To examine the question of the imperfection in the paper that Morgan notes, it is useful to go to the original paper. The problem that Bayes considered and Morgan paraphrased reads:

> Given the number of times in which an unknown event has happened and failed: Required the chance that the probability of its happening in a single trial lies somewhere between any two degrees of probability that can be named.

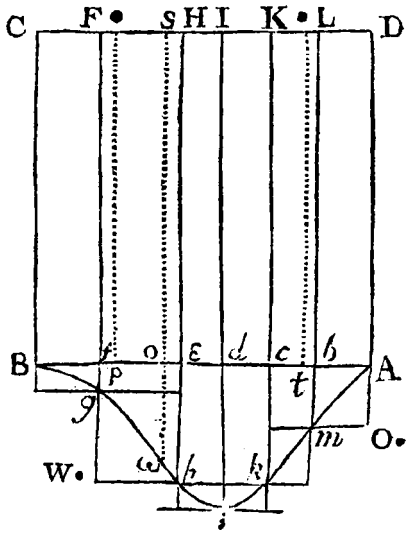


FIG. 14. *Bayes's model table.*

Bayes solved this problem by considering the model table shown in Figure 14. A ball labeled W is thrown across the table in such a way that it is equally likely to come to rest anywhere on the table. Through the point that it comes to rest on the table, draw the line os. Then throw the ball labeled O n times and count the number of times it falls on either side of the line. These are the successes and failures. Under this physical model one can now find the chance that the probability of success θ is between two given numbers f and b. On using Stigler's (1986) notation, this probability is given by

$$P(f < \theta < b|X = x) = \frac{\int_f^b \binom{n}{x} \theta^x (1-\theta)^{n-x}\, d\theta}{\int_0^1 \binom{n}{x} \theta^x (1-\theta)^{n-x}\, d\theta}.$$

The evaluation of the numerator is the difference between two incomplete beta functions. Approximations to this function can be obtained through an infinite series when x is small. Bayes did this in Rule 1 of his essay. At that point Price writes, "Thus far Mr. Bayes' essay." After that there appear two other rules in the essay for the approximation to the integral. Stigler (1986, page 130), in his commentary on Bayes' work, rightly notes that when x and $n - x$ are not small, the computational problem is "a formidable one." Later Price (1764) gave a full transcription of one of Bayes' derivations of the second rule to approximate the incomplete beta function and then provided his own improved approximation. The "imperfect" solution to the problem was in the practical application of it, and that required the approximations to the integral. Despite Price's efforts at an improved solution in 1764, about 50 years later Thomson (1812), in a history of the Royal Society, described the results of the paper and then stated, "The solution is much too long and intricate to be of much practical utility." That Bayes' notebook is now with the Equitable Life Assurance Society is due in all probability to William Morgan. The notebook was probably among the papers that Price obtained from the Bayes family. In the 1760s, Price became interested in the development of actuarial techniques for insurance and annuities. For example, in 1768 Price was asked by Equitable's actuary, John Edwards, to provide advice on the pricing of a reversionary annuity (Price, 1983, pages 56 and 277–283). The ties with Equitable Life soon became much closer. Price's nephew and biographer William Morgan, through Price's influence, was hired at Equitable Life as an assistant actuary in 1774 (*Dictionary of National Biography*). The next year Morgan became an actuary at Equitable and remained in that position until his retirement in 1830. Upon his retirement, Morgan's position was taken by his son Arthur Morgan, who remained chief actuary of Equitable until his own retirement in 1870. William Morgan and Richard Price were collaborators on actuarial problems and publications. As Price's biographer, Morgan probably had Price's papers and Bayes' notebook in his possession. For whatever reason, the notebook was left at Equitable's offices, probably by William Morgan, where it surfaced in the 1940s in the office of Maurice Ogborn, another Equitable actuary. While working one day at the Royal Society, Ogborn noticed that the handwriting in the notebook appeared to be the same as Bayes' handwriting on other documents at the Royal Society (Perks, 1947, page 318). Now identified, it remains today at Equitable Life.

How Bayes and Price, who was about 22 years Bayes' junior, became friends is unknown. That Bayes probably and Price definitely held Arian views with regard to theology is not enough to establish a relationship. There are a few possibilities to speculate upon. Some of Price's theological writing was influenced by the debate over the source of God's action in the world as described in Section 6. His theological work, *A Review of the Principal Questions and Difficulties in Morals*, which touches on the same issues, was first published in 1758 (Price, 1948). Although he disagreed with Bayes' position (Thomas, 1977, pages 72, 81 and 82), Bayes' theological work may have motivated Price to seek out Bayes to discuss Bayes' views on the subject. Furthermore, they had mutual friends or acquaintances through whom they could have met. Price prepared for the ministry at

the Fund Academy in Tenter Alley under John Eames (Thomas, 1977, page 10) and Eames was one of Bayes' sponsors in his election to the Royal Society. Price also knew John Ellicott and John Canton, both of whom had connections to Bayes. Ellicott was one of the sponsors for Price's election to the Royal Society in 1765 (Thomas, 1977, page 134); Ellicott worked with Eames and also made Bayes' gold watch. It was to Canton that Price submitted Bayes' paper referring to Bayes as "our deceased friend"; in 1755 Bayes had been corresponding with Canton on the work of Simpson (1755).

Many people have written in detail about the results in Bayes (1763a). Excellent treatments can be found in Stigler (1986) and Dale (1991a), so no attempt will be made here to repeat their efforts. Instead, beyond what I have given as Price's motivation for working on the paper, I will just touch only on the model table. Stigler (1986) has rightly pointed out that Bayes never specified what type of table he had in mind, although many, including Fisher and Pearson, "have promoted it to a billiard table." At first glance a billiard table makes sense because of the side cushions; balls rolled across the table would come to rest somewhere on the table rather than falling off. The major problem with a billiard table as the model is that it has pockets, which do not appear in Figure 14. However, some billiard tables from the early 17th century, which did have side cushions, did not have pockets. Illustrated in Figure 15 is one such table that appears in an algebra text (de Graaf, 1672). Near the end of the 17th century, billiard tables with pockets were common. This type of table is described in detail in R. H. (1684). It is interesting to note that the anonymous author R. H. stated that the table needs to be carefully constructed and balanced so that "... your Ball may run true upon any part of the Table, without leaning or declining to one side." This is essentially the kind of behavior Bayes is modeling in his paper. A possible candidate for Bayes' billiard table is the one at the stately home Knole in Sevenoaks near Tunbridge Wells. Knole appears in various Tunbridge Wells guides as a recommended place to visit. It is possible that Bayes visited Knole; it would be almost certain that Stanhope would have visited there. The major problem with the conjecture that Bayes' model table was the billiard table at Knole is that the table has been upgraded to include pockets at some time, which is presently unknown. Sackville West (1906, page 41), who was a resident at Knole, briefly described the billiard room and its contents:

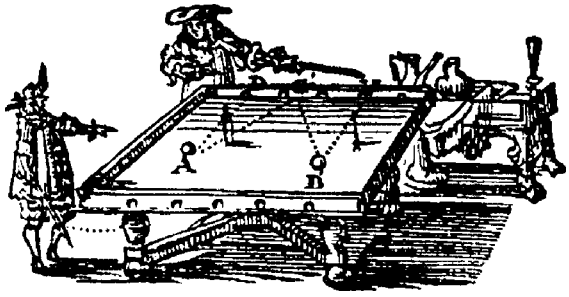


FIG. 15. *An early billiard table without pockets.*

> This room, which is really part of the Leicester Gallery, contains a billiard table, the lower structures of which are evidently of the time of Charles I, while the top or bed, with cushions and pockets are of a later date.

10. DECLINE AND DEATH

I have speculated that Bayes' decision over the years 1749–1752 to leave the pulpit at Mount Sion Chapel in Tunbridge Wells may have stemmed from his Arian theology. It may also have been due to ill health. An entry near the end of Bayes' notebook is for a prescription. The entry occurs several pages after a transcription of an article from *London Magazine* written in 1750, so the prescription was probably obtained after that date. The prescription was transcribed by Dale (n.d.) and interpreted as follows:

> ... take two drachms of the powder seed of cumin and chamomile (both carminative); 1/2 drachm of a solution of calcium carbonate; 1 scruple of camphor, in a solution of 2 drachms of spirits of turpentine (turpentine oil is a rubefacient used in liniments for rheumatic pain and stiffness); mix together, to one fluid ounce, an unguent of 3 fluid ounces of sambucus (the elder flower—has an astringent action on the skin) with 1 fluid ounce of saponis nigra (liquid drawn from lye soap—an emulsifying agent). Mix and use as a liniment. This is probably a prescription for stiffness of the joints or rheumatism.

The prescription could have been used for other purposes; liniments were used in the 18th century for a variety of ailments. The ailment could also have been arthritis or gout. However, Dale's interpretation does have support in the 18th century medical literature. On

checking many of the ingredients of the prescription in James (1743–1745) there are several uses for each ingredient. What seems common to the ingredients taken together is pain relief, relief of inflammation and alleviation of rheumatism or gout. Used externally, cumin was recommended for pains in the chest or side; likewise chamomile. When used in a plaster or liniment, chamomile was also recommended to relieve inflammation, as was camphor. In a liniment or unguent, sambucus was used to treat rheumatism or pain from gout. One only hopes that Bayes did not receive the full treatment for rheumatism. The treatment, described by James (1746, page 188), involved blood letting, blistering of the skin and laxatives, in addition to the more reasonable regimen of tepid baths, rest in a warm bed and the application of liniments.

In 1755 Bayes was definitely ill, perhaps seriously. After Stanhope had asked him to look over a paper by Murdoch, Bayes wrote back on April 25, "Mr. Murdockes observations coming to hand when I was not well has been one reason, that they have been so long detained here." Bayes' illness may have been periodic. He signed his will December 12, 1760 and died less than 4 months later on April 7, 1761. He was probably in ill health at the time he signed his will. From newspaper reports about his death that appear in *The Public Advertiser* and *Whitehall Evening Post*, the only information about his death is that he died suddenly. Although, as in the rheumatism, there may be several explanations, a likely cause of death was heart attack. The heart attack would be consistent with rheumatism, especially acute rheumatism (Copeman, 1964, page 126). Rheumatism also may have been the reason he stayed in Tunbridge Wells after his retirement from the ministry rather than move to London where his siblings were situated. Although many people came to Tunbridge Wells for social reasons, some came there at least partially for their health, including Elizabeth Montagu and Philip Dormer Stanhope, Earl Chesterfield.

After his death in Tunbridge Wells, Bayes' body was taken to Founders' Hall in London, which Nonconformists had used as a meetinghouse for Scots Presbyterians since 1700 (Wilson, 1808–1814, Vol. 2, page 293). Bayes had requested in his will that his funeral expenses be as frugal as possible. In a typical Nonconformist funeral (Davies, 1961, pages 47 and 136) there would be a funeral procession from the place where the body had been kept, normally the deceased's home, to the cemetery. The Bunhill Fields registers show, for example, that John Bayes was brought from his brother Samuel's home in Cheapside in 1743, that Joshua Bayes was brought from the same place in 1746 and that Nathaniel Bayes was brought from his home in Snow Hill in 1764. Although Nonconformists objected to it for Sunday services, the *Book of Common Prayer* might be used for the burial service at the grave; there was no burial service in the chapel. Boys might sing at the graveyard or perhaps during the funeral procession. How the chapel came to play a part in the funeral was that a funeral sermon might be preached on the Sunday following the burial. The pulpit and clerk's desk would be hung with black cloth and the galleries would be hung with black baize. The preacher would not wear his hat during the funeral sermon, but the hat would be adorned with a silken token of mourning. Bayes' desire for frugality meant that there was no funeral sermon. Presumably the preacher was given a fee for the funeral sermon and there was also the cost of publication of the sermon. That the funeral was, however, not as frugal as possible comes from Nathaniel Bayes' 1764 will in which he wrote, "I desire to be interred in the same decent manner as my Brothers were but not to have any Boys to sing at my funeral." Bayes was buried in the family vault along with his parents and siblings in Bunhill Fields Cemetery in London (Figure 16). The only recorded expense for his funeral was 14 shillings to open the vault. This was the same expense incurred at the funerals of his father and brothers.

FIG. 16. *The Bayes family vault, Bunhill Fields.*

11. POSTSCRIPT

There are comments that can be made on two outstanding questions regarding Thomas Bayes: What was he like as a person and what did he look like?

On the first question, Holland (1962) concluded, based on his own research, that Bayes was a "quiet man, of earnest thought and abiding faith and of immense intellectual stature. . . ." My own assessment differs somewhat from Holland. Based on the fact that he denied the pulpit to other Nonconformists, especially on Easter Day, says something of his strength of resolve rather than his quietness. His work as a referee or critic of other mathematical research, which was not done anonymously, at least in the case of Patrick Murdoch, shows a certain confidence in his own abilities and work. As a critic of others, he was very insightful. His intellectual stature may not have been immense, but he was recognized as an excellent mathematician. Finally, his faith may not have been wholly abiding if he drifted from orthodox Presbyterianism to Arianism. If indeed he took on Arian beliefs, it shows a willingness to question his faith seriously despite the fact that his father, and probably his siblings, remained orthodox.

The only known portrait that is possibly of Bayes appeared in a book by O'Donnell (1936, page 335); see Figure 17. Unfortunately, there are many possible anachronisms in the picture that would lead one to doubt that it is a true portrait. Let me repeat, with some additions, what I once wrote in *The IMS Bulletin* in response to a contest to identify the person in the picture. Note in the picture the apparent absence of a wig; or if a wig is present, it is the wrong style for the period. The formal portraits of Joshua Bayes, Philip Doddridge and Richard Price all have their subjects in wigs. Bayes most likely would have worn a wig in the style of his contemporary, Philip Doddridge. In the portraits of Whiston and Simpson, the subjects are not wearing wigs. However, Simpson was not a clergyman and Whiston is also not wearing his ecclesiastical garments (instead it appears to be a frock coat), which might be expected in a formal portrait. The gown, or possibly a frock coat, worn in the picture of Bayes appears to have a high collar, which is anachronistic. Doddridge, in his portrait, appears to be wearing a gown; however, the style is different from that worn by Bayes. If Bayes' accoutrement is a frock coat, then it should be tightly fitting as in the portraits of Price and Whiston. Finally, Bayes appears to be wearing a clerical collar with bands. Note that the bands seem to come out at the bottom of the collar, while in the portraits of Berkeley, Doddridge, Price, Whiston and Bayes' father Joshua, the bands come over the top of the collar. For a general discussion of the costumes of the 18th century, see Cunnington and Cunnington (1964). Beyond the problems of possible anachronistic style, there is the question of the origin of this portrait. O'Donnell (1936) provided no clue. It is known that the portrait of Joshua Bayes was donated to Dr. Williams Library in 1799 by Joshua's grandson, Bayes Cotton (Jeremy, 1885, page 124). As noted in Section 3, the major beneficiary of the various estates of the Bayes family turned out, in the end, to be Bayes Cotton. Since he possessed the portrait of his grandfather Joshua, he would be the likely possessor of any portrait of his uncle Thomas. In view of the fact that in 1799 no donation was made of Thomas Bayes' portrait, it is possible that no portrait existed within the family. The available evidence puts up too many questions to conclude that the one known picture is a true likeness of Bayes.

FIG. 17. *Thomas Bayes*?

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RESEARCH SOURCES

Manuscripts

British Library Manuscripts
- Letters to Thomas Bayes and John Skinner Smith in Ward's Latin Correspondence. Additional Manuscript 6224, p. 116.
- List of persons educated under John Ward from 1715 to 1731. Additional Manuscript 6181.f.111.

Centre for Kentish Studies
- John Bowra, 1738 map. A Survey of Tunbridge Wells and All Places of Note Within a Mile and an Half of the Chappel.
- Stanhope of Chevening Manuscripts: U1590/C21—Papers by several eminent mathematicians addressed to or collected by Lord Stanhope (contains two sets of papers and correspondence with Thomas Bayes); U1590/C14/2—Correspondence with P. Murdoch (contains two letters by Murdoch commenting on the material sent by Bayes to him through Stanhope).
- Quarter Session Records Q/SB 1721, 1751, 1752.

Dr. Williams Library
- John Evans List of Dissenting Congregations 1715–1729 MS 34.4.
- Minute books of the Body of Protestant Dissenting Ministers of the Three Denominations in and about the Cities of London and Westminster **1** 1727–1761.

Farthing, Roger
- Typescript letters written by Roger Farthing of Tunbridge Wells dated 1988 commenting on 69 London Road in Tunbridge Wells based on uncatalogued (at the time) deeds of the Buss Stone and Co. bearing the archive number U2737.

Guildhall Library, London
- London directories.

Lambeth Palace Library
- Vicar-General Marriage Licence Allegations (the Bayes–Carpenter marriage licence appears in the index; the original has been lost).

Lincoln's Inn
- *The Records of the Honorable Society of Lincoln's Inn. Vol. I. Admissions. From A.D. 1420 to A.D. 1799.* Lincoln's Inn, 1896.
- *The Records of the Honorable Society of Lincoln's Inn. The Black Books. Vol. III. From A.D. 1660 to A.D. 1775.* Lincoln's Inn, 1899.

Public Record Office
- Chancery Proceedings C 11/1636/3.
- Nonconformist Registers: Leather Lane Holborn, RG 4/4052; St. Thomas Chapel Southwark, RG 4/4489; Mount Zion Chapel, Tunbridge Wells, RG 4/1981.
- Bunhill Fields Burial Registers: 1741–1747, RG 4/3980; 1754–1761, RG 4/3982; 1761–1766, RG 4/3983.
- Prerogative Court of Canterbury Wills (now in Family Records Centre): John Bayes, 1743, PROB 11/729/329; Joshua Bayes, 1746, PROB 11/746/110; Nathaniel Carpenter, 1753, PROB 11700/74; Thomas West, 1754, PROB 11/809/185; Thomas Bayes, 1761, PROB 11/865/158; Nathaniel Bayes, 1764, PROB 11/904/454; Mary Bayes, 1780, PROB 11/1061/57; Samuel Bayes, 1789, PROB 11/1183/485.

Royal Society
- MS 790, undated letter from Philip Stanhope to Martin Folkes in the associated letters of Marin Folkes 1704–1744.
- Canton Papers, Correspondence, Volume 2, folio 32. Two letters from Bayes to Canton, one on electricity and the other on probable error of the mean.
- Miscellaneous Manuscripts, Volume I, No. 17. Letter from Bayes to Canton on a result of Simpson.
- Cert I, 210. Nomination certificate of Thomas Bayes.
- Journal Books of Scientific Meetings, Volume 17, November 30, 1739–November 30, 1742.

The Equitable Life Assurance Society
- Thomas Bayes' notebook.

University of Edinburgh Library
- Commonplace Book of Professor Charles Mackie. Alphabetical list of those who attended the Prelections on History and Roman Antiquitys from 1719 to 1744 Inclusive. Collected 1 July 1746.
- Leges Bibliothecae Universitatis Eninensis. Names of persons admitted to the use of the library
- Library Accounts 1697–1765
- List of students who attended the Divinity Hall in the University of Edinburgh from 1709 to 1727. Copied from the MSS of the Revd. Mr. Hamilton, then Professor of Divinity.
- List of theologues in the University of Edinburgh 1709–1724.
- The Morgan Transcripts of the Matriculation Albums of the University of Edinburgh. Scholarium Matricula ab Anno MDCCIV

Newspapers and Periodicals

Gentleman's Magazine (1761).
The IMS Bulletin **17** (1988) 276–278.
London Magazine (1749).
The Journal of the Presbyterian Historical Society of England **2** (1920–1923) 222–224.
Protestant Dissenter's Magazine **5** (1798).
The Public Advertiser, April 13, 1761.
Public Characters **4** (1800–1801). Richard Phillips, London.
Scots Magazine **3** (1741) 371–374.
Whitehall Evening Post, April 13, 1761.

REFERENCES

(Questionable authorship is indicated by square bracketted names.)

AINSWORTH, R. (1720). *Monumenta Vetustatis Kempiana, ex Vetustis Scriptoribus Illustrata, Eosque Vicissim Illustrantia*. Daniel Bridge, London.

ANDERSON, J. G. (1941). The Reverend Thomas Bayes, F.R.S. *Mathematical Gazette* **25** 160–162.

ANONYMOUS (1986). *Index to American Periodicals of the 1700's: Keyed to University Microfilms APS 1*. Computer Indexed Systems, Indianapolis, IN.

ARCHER, J. (1720). *A Sermon at the Opening of the New-Chappel at Tunbridge-Wells, Aug. 1, 1720*. Richard Ford, London.

BARNARD, G. A. (1958). Thomas Bayes—a biographical note. *Biometrika* **45** 293–295.

BARTON, M. (1937). *Tunbridge Wells*. Faber and Faber, London.

[BAYES, T.] (1731). *Divine Benevolence, or, an Attempt to Prove that the Principal End of the Divine Providence and Government is the Happiness of his Creatures*. John Noon, London.

[BAYES, T.] (1736). *An Introduction to the Doctrine of Fluxions, and a Defence of the Mathematicians Against the Objections of*

the Author of the Analyst, so far as They Are Designed to Affect their General Methods of Reasoning. John Noon, London.

BAYES, T. (1763a). An essay towards solving a problem in the doctrine of chances. *Philosophical Transactions* **53** 370–418. [Reprinted (1958) Studies in the history of probability and statistics IX. Thomas Bayes's essay towards solving a problem in the doctrine of chances. *Biometrika* **45** 296–315 and (1970) E. S. Pearson and M. G. Kendall, eds. *Studies in the History of Statistics and Probability*. Griffin, London.]

BAYES, T. (1763b). A letter from the late Reverend Mr. Thomas Bayes, F.R.S. to John Canton, M.A. and F.R.S. *Philosophical Transactions* **53** 269–271.

BELLHOUSE, D. R. (2002). On some recently discovered manuscripts of Thomas Bayes. *Historia Math.* **29** 383–394.

BERKELEY, G. (1734). *The Analyst*; *or, a Discourse Addressed to an Infidel Mathematician*. J. Tonson, London.

BIRCH, T. (1766). *An Account of the Life of John Ward, LL.D., Professor of Rhetoric in Gresham College*; *F.R.S. and F.S.A.* P. Vaillant, London.

BROWNE, J. (1877). *History of Congregationalism, and Memorials of the Churches in Norfolk and Suffolk*. Jarrold and Sons, London.

BURR, T. B. (1766). *The History of Tunbridge–Wells*. Hingeston, London.

CAJORI, F. (1919). *A History of the Conceptions of Limits and Fluxions in Great Britain from Newton to Woodhouse*. The Open Court Publishing Co., Chicago and London.

CHALMERS, A. (1812–1817). *The General Biographical Dictionary*: *Containing an Historical and Critical Account of the Lives and Writings of the Most Eminent Persons in Every Nation*; *Particularly the British and the Irish*; *from the Earliest Accounts to the Present Time*. J. Nichols & Son, London.

CLAY, J. W., ed. (1894–1895). *Familiae Minorum Gentium* **1**–**3**. Harleian Society Publications **37**–**39**. London.

CLIMENSON, E. J. (1906). *Elizabeth Montagu, the Queen of the Blue-Stockings*: *Her Correspondence from 1720 to 1761* **1**, **2**. John Murray, London.

COOMER, D. (1946). *English Dissent under the Early Hanoverians*. Epworth Press, London.

COPEMAN, W. S. C. (1964). *A Short History of the Gout and the Rheumatic Diseases*. Univ. California Press, Berkeley.

CUNNINGTON, C. W. and CUNNINGTON, P. (1964). *Handbook of English Costume in the Eighteenth Century*, 2nd ed. Faber and Faber, London.

DALE, A. I. (n.d.). On a notebook attributed to Thomas Bayes. Manuscript seen courtesy of the author.

DALE, A. I. (1990). Thomas Bayes: Some clues to his education. *Statist. Probab. Lett.* **9** 289–290.

DALE, A. I. (1991a). *A History of Inverse Probability from Thomas Bayes to Karl Pearson*. Springer, New York.

DALE, A. I. (1991b). Thomas Bayes' work on infinite series. *Historia Math.* **18** 312–327.

DALZEL, A. (1862). *History of the University of Edinburgh from its Foundation* **1**, **2**. Edmonston and Douglas, Edinburgh.

DAVIES, H. (1961). *Worship and Theology in England* **3**. *From Watts and Wesley to Maurice, 1690–1850*. Princeton Univ. Press.

DE GRAAF, A. (1672). *De Beginelen van de Algebra of Stelkonst, Volgens de Manier van Renatus Des Cartes*. Jan Rieuwerstz, Amsterdam.

DE MOIVRE, A. (1756). *The Doctrine of Chances*, 3rd ed. Millar, London.

DODDRIDGE, P. (1739–1756). *The Family Expositor*: *Or, a Paraphrase and Version of the New Testament*; *with Critical Notes*; *and a Practical Improvement of Each Section*. Richard Hett, London.

DODDRIDGE, P. (1822). *A Course of Lectures on the Principal Subjects in Pneumatology, Ethics, and Divinity*; *with References to the Most Considerable Authors on Each Subject, Volume the First* (with notes and references by A. Kippis). Ogle, Duncan and Co., London.

EDWARDS, A. W. F. (1993). Thomas Bayes. In *Dictionary of National Biography*: *Missing Persons* (C. S. Nicholls, ed.) 50–51. Oxford Univ. Press.

EVANS, G. E. (1897). *Vestiges of Protestant Dissent*. F. and E. Gibbons, Liverpool.

FISHER, T. (1857). Thomas Bayes. *The Imperial Dictionary of Universal Biography* **1**. William Mackenzie, London.

FOSTER, J. (1752). *Discourses on All the Principal Branches of Natural Religion and Social Virtue* **2**. Noon, London.

FRANKLIN, B. (1959). *The Papers of Benjamin Franklin* (L. W. Labaree and W. J. Bell, Jr., eds.). Yale Univ. Press, New Haven, CT. (Volumes 10–17 cover the years 1762–1770.)

GOLDSTINE, H. H. (1977). *A History of Numerical Analysis from the 16th through the 19th Century*. Springer, New York.

GRANT, A. (1884). *The Story of the University of Edinburgh During Its First Three Hundred Years*. Longmans, Green and Co., London.

GROSVENOR, B. (1723). *A Sermon on the Occasion of the Death and Funeral of John Deacle, Esq., Who Departed this Life, Oct 29. Preached at Crosby Square Nov. 10, 1723*. John and Barham Clark, London.

GUICCIARDINI, N. (1989). *The Development of Newtonian Calculus in Britain 1700–1800*. Cambridge Univ. Press.

HACKING, I. (1970–1980). Thomas Bayes. In *Dictionary of Scientific Biography* (C. C. Gillespie, ed.). Scribner, New York.

HALLEY, R. (1869). *Lancashire*: *Its Puritanism and Nonconformity* **2**. Tubbs and Brook, Manchester.

HOADLY, B. and WILSON, B. (1756). *Observations on a Series of Electrical Experiments*. London.

HOLLAND, J. D. (1962). The Reverend Thomas Bayes, F.R.S. (1702–61). *J. Roy. Statist. Soc. Ser. A* **125** 451–461.

HOME, R. W. (1974). Some manuscripts on electrical and other subjects attributed to Thomas Bayes, F.R.S. *Notes and Records Roy. Soc. London* **29** 81–90.

HUME, D. (2000). *An Enquiry Concerning Human Understanding*: *A Critical Edition* (T. L. Beauchamp, ed.). Clarendon, Oxford.

JAMES, R. (1743–1745). *A Medicinal Dictionary*: *Including Physic, Surgery, Anatomy, Chymistry, and Botany, in all their Branches Relative to Medicine*. T. Osborne, London.

JAMES, R. (1746). *The Modern Practice of Physic*; *As Improv'd by the Celebrated Professors, H. Boerhaave and F. Hoffman, Physicians to the Late and Present King of Prussia*. J. Hodges, London.

JAMES, T. S. (1867). *The History of the Litigation and Legislation Respecting Presbyterian Chapels and Charities in England and Ireland between 1816 and 1849*. Hamilton Adams and Co., London.

JEREMY, W. D. (1885). *The Presbyterian Fund and Dr. Daniel Williams's Trust*: *With Biographical Notes of the Trustees, and*

Some Account of Their Academies, Scholarships and Schools. Williams and Norgate, London.

JESSEPH, D. M. (1993). *Berkeley's Philosophy of Mathematics*. Univ. Chicago Press.

JOHNSTON, W. (1771). *The Improved Christian's Courage and Comfort, in Afflictions and Death; through the Gracious Presence, and Influence of his Heavenly Shepherd. Illustrated in a Funeral Sermon, Preached at Tunbridge-Wells, on the 23d of December, 1770*. London.

JONES, J. A., ed. (1849). *Bunhill Memorials*. James Paul, London.

LAGRANGE, J. (1869–1870). *Œurvres de Lagrange, Publiées par les Soins de M. J.-A. Serret* **3** (1869) 441–476; **5** (1870) 663–684. Gauthier-Villars, Paris.

LEADER, J. D. (1897). *The Records of the Burgery of Sheffield, Commonly Called the Town Trust, with Notes and Introduction by John Daniel Leader*. Elliot Stock, London.

LEADER, R. E. (1905–1906). *History of the Company of Cutlers in Hallamshire in the County of York* **1**, **2**. Pawson and Brailsford, Sheffield.

LILLYWHITE, B. (1963). *London Coffee Houses*: *A Reference Book of Coffee Houses of the Seventeenth, Eighteenth and Nineteenth Centuries*. Allen and Unwin, London.

LOCKE, J. (1975). *An Essay Concerning Human Understanding* (P. Nidditch, ed.). Clarendon, Oxford.

MACLAURIN, C. (1742). *A Treatise of Fluxions*: *In Two Books, by Colin MacLaurin*. T. W. and T. Ruddimans, Edinburgh.

MATTHEWS, A. G. (1934). *Calamy Revised*: *Being a Revision of Edmund Calamy's Account of the Ministers and Others Ejected and Silenced 1660–1662*. Clarendon, Oxford.

MILLS, B. (1733). *A Funeral Sermon Occasioned by the Death of the Late Reverend Mr. John Archer, Who Departed the Life, September 23, 1733. Preached at Tunbridge-Wells, September 30*. R. Hett, London.

MONTAGU, E. (1809–1813). *The Letters of Mrs. Elizabeth Montagu, with Some of the Letters of her Correspondents* **1**–**4**. T. Cadell and W. Davies, London. (Reprinted 1974 by AMS Press, New York.)

MORGAN, W. (1815). *Memoirs of the Life of the Rev. Richard Price, D.D., F.R.S.* R. Hunter and R. Rees, London.

MURCH, J. (1835). *A History of the Presbyterian and General Baptist Churches in the West of England; with Memoirs of Some of Their Pastors*. Hunter, London.

NEWMAN, A. (1969). *The Stanhopes of Chevening*: *A Family Biography*. Macmillan, London.

NEWTON, I. (1969). *Mathematical Principles of Natural Philosophy* (R. Thorp, trans.). Dawsons, London.

NUTTALL, G. F. (1979). *Calendar of the Correspondence of Philip Doddridge, D.D. (1702–1751)*. H. M. Stationery Office, London.

O'DONNELL, T. (1936). *History of Life Insurance in Its Formative Years*; *Compiled from Approved Sources*. American Conservation Co., Chicago.

[ONELY, R.] (1771). *A General Account of Tunbridge Wells and Its Environs*: *Historical and Descriptive*. G. Pearch, London.

PEARCE, L. (1904). *Historical Association of the Free Churches of Tunbridge Wells 1642–1904*. Pearce and Co., Tunbridge Wells.

PEARSON, K. (1978). *The History of Statistics in the 17th and 18th Centuries against the Changing Background of Intellectual, Scientific and Religious Thought* (E. S. Pearson, ed.). Griffin, London.

PEMBERTON, H. (1728). *A View of Sir Isaac Newton's Philosophy*. S. Palmer, London.

PERKS, W. (1947). Some observations on inverse probability including a new indifference rule (with discussion). *J. Inst. Actuar.* **73** 285–334.

PHIPPEN, J. (1840). *Colbran's New Guide for Tunbridge Wells, Being a Full and Accurate Description of the Wells and Its Neighbourhood, within a Circuit of Nearly Twenty Miles*. John Colbran, Tunbridge Wells.

PRICE, R. (1764). A demonstration of the second rule in the essay towards the solution of a problem in the doctrine of chances, published in the *Philosophical Transactions*, Vol. LIII. *Philosophical Transactions* **54** 296–325.

PRICE, R. (1767). *Four Dissertations. I. On Providence. II. On Prayer. III. On the Reasons for Expecting that Virtuous Men Shall Meet after Death in a State of Happiness. IV. On the Importance of Christianity, the Nature of Historical Evidence, and Miracles*. A. Millar and T. Cadell, London.

PRICE, R. (1769). Observations on the expectation of lives, the increase of mankind, the influence of great towns on population, and particularly the state of London with respect to healthfulness and number of inhabitants. *Philosophical Transactions* **59** 89–125.

PRICE, R. (1948). *A Review of the Principal Questions in Morals* (D. Daiches Raphael, ed.). Clarendon, Oxford. (This publication gives the text of the third edition of 1787 together with an introduction.)

PRICE, R. (1983). *The Correspondence of Richard Price* **1**: *July 1748–March 1778* (D. O. Thomas and W. B. Peach, eds.). Duke Univ. Press, Durham, NC.

R. H. (1684). *The School of Recreation, or, The Gentlemans tutor*. H. Rodes, London.

SACKVILLE WEST, L. (1906). *Knole House*: *Its State Rooms, Pictures and Antiquities*. J. Salmon, Sevenoaks.

SIMPSON, T. (1755). A letter to the Right Honourable George Earl of Macclesfield, President of the Royal Society, on the advantage of taking the mean of a number of observations, in practical astronomy. *Philosophical Transactions* **49** 82–93.

SMITH, G. C. (1980). Thomas Bayes and fluxions. *Historia Math.* **7** 379–388.

SPRANGE, J. (1780). *A Description of Tunbridge Wells, in Its Present State*; *and the Amusements of the Company, in the Time of the Season*; *and of the Ancient and Present State of the Most Remarkable Places, in the Environs, Comprehending a Circuit of about Sixteen Miles Round the Place*. Sprange, Tunbridge Wells.

STEELE, R. J. (1973). *Sources for Nonconformist Genealogy and Family History*. Society of Genealogists, London.

STEPHEN, L. and LEE, S., eds. (1921–1922). *Dictionary of National Biography*. Oxford Univ. Press, London.

STIGLER, S. M. (1986). *The History of Statistics*: *The Measurement of Uncertainty before 1900*. Harvard Univ. Press.

STRANGE, C. H. (1949). *Nonconformity in Tunbridge Wells*. Courier Printing and Publishing Co., Tunbridge Wells.

SUMMERS, W. H. (1905). *History of the Congregational Churches in the Berks, South Oxon and South Bucks Association, with Notes on the Earlier Nonconformist History of the District*. W. J. Blacket, Newbury.

TAYLOR, B. (1715). *Methodus Incrementorum Directa et Inversa*. Innys, London.
THOMAS, D. O. (1977). *The Honest Mind*: *The Thought and Work of Richard Price*. Clarendon, Oxford.
THOMSON, T. (1812). *History of the Royal Society, from Its Institution to the End of the Eighteenth Century*. Robert Baldwin, London.
TIMPSON, T. (1859). *Church History of Kent*: *From the Earliest Period to the Year MDCCCLVIII*. Ward and Co., London.
TOULMIN, J. (1814). *An Historical View of the State of the Protestant Dissenters in England, and of the Progress of Free Enquiry and Religious Liberty, from the Revolution to the Accession of Queen Anne*. Richard Cruttwell, Bath.
WARD, J. (1740). *The Lives of the Professors of Gresham College, to which Is Prefixed the Life of the Founder Sir T. Gresham*. London.
WEINREB, B. and HIBBERT, C., eds. (1983). *The London Encyclopaedia*. Macmillan, London.
WHISTON, W. (1749). *Memoirs of the Life and Writings of Mr. William Whiston. Containing Memoirs of Several of His Friends Also*. Printed for the author, London.
WILES, M. (1996). *Archetypal Heresy*: *Arianism through the Centuries*. Clarendon, Oxford.
WILLIAMS, P. O. (1922). The Founders' Hall meeting. *J. Presbyterian Historical Society of England* **2** 133–138.
WILSON, G. (1882). *Memorials of the Old Ministers*: *Founders' Hall, London Wall, Canonbury*. Hodder and Stoughton, London.
WILSON, W. (1808–1814). *The History and Antiquities of Dissenting Churches and Meeting Houses in London, Westminster and Southwark*; *Including the Lives of Their Ministers, from the Rise of Nonconformity to the Present Time* **1**–**4**. London.

Comment

Andrew Dale

INTRODUCTION

Although, as Bellhouse has mentioned, some research into the life and works of Thomas Bayes was undertaken in the first half (roughly) of the 20th century, interest seems to have increased over the past few decades, and Bellhouse has contributed to the satisfaction of such interest in no small way. His recent discovery of manuscripts relating to Bayes in the Stanhope of Chevening collection in the Kent County Archives in Maidstone provides an important addition to Bayesiana, and his sharing of this and other information with the wider statistical community in this article is most welcome. I propose here merely to add a few biographical remarks to supplement the details provided by Bellhouse.

ARIANISM AND PRESBYTERIANISM

Thomas Bayes was a Presbyterian minister. English Presbyterianism, certainly in his time, differed from the Scottish version we know today. The former was favored during the reign of Edward VI by leading English ecclesiastics as a form of church government and discipline. It was later suppressed by Elizabeth, who found it to be incompatible with absolute monarchy. After the Civil War, Puritanism began to show Presbyterian tendencies and it was soon deemed worthy of suppression. (The Presbyterian Church differs from the Church of England in recognizing only one spiritual order, that of *presbyter*, as opposed to the latter's bishops, priests and deacons.)

In the article on Presbyterianism in the 14th edition of the *Encyclopædia Britannica* (Vol. 18, page 444) we find

> From the beginning of the 18th century the greater number of the Presbyterian congregations became practically independent in polity and Unitarian in doctrine.

Although Arianism was not Unitarianism, its adherents occupying a position somewhere between that of the Unitarians and that of the Trinitarians, we should, in the light of this remark, not be surprised at Bayes' supposed Arianism and the eventual connection of his Mount Sion congregation with the Independents.

However, to say that Bayes was an Arian does not completely describe his religious beliefs. Edward Gibbon, in Chapter 21 of his *The History of the Decline and Fall of the Roman Empire*, instances 18 creeds that were all at one time viewed as Arian. Despite these variations, Arianism broadly speaking demanded of its adherents the acceptance of a relationship of distinction and subordination between the three persons of the Trinity unlike that recognized by the established church.

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One might find further confirmation, albeit very slight, of Bayes' supposed Arianism in noting that he quotes Dr. Samuel Clarke's *Sermons* in his *Divine Benevolence*. Clarke, well known to be an Arian, examined 1257 Biblical texts in his *Scripture Doctrine of the Trinity*, concluding that the Father was alone supreme, the Son was divine only inasmuch as divinity is communicable by the supreme God, and the Holy Spirit was inferior to both the Father and the Son, not only in order, but also in dominion and authority (see Hastings, 1971, Vol. 1, page 786).

BAYES' FRIENDS AND RELATIONS

Bellhouse follows most recent writers in placing Samuel Bayes in Manchester after his ejection. However, I suggest that there is perhaps room for some doubt. Turner (1911) listed contemporary records of Samuel Bayes in association with Sankey (license to preach, meetinghouse). He also refers to one Samuel Buze as being licensed to preach and teach in his house in Manchester. Matthews (1934) wrote, in reference to Samuel Bayes, "Licensed (P.), as of Sankey, Lancs., 5 Sep. 1672; also, as Buze, at his house, Manchester." Are Buze and Bayes the same or, like Lord Russell of Liverpool and Bertrand, Earl Russell, is neither of them the other? I do not know.

After their marriage, Joshua and Ann Bayes possibly lived for some time in Bovingdon, Hertfordshire. Urwick (1884) recorded that the first trust deed for the ("lately-erected") Nonconformist chapel in Box Lane is dated 1697. In terms of this deed the chapel was transferred by the proprietors, Thomas and Mary Lomax, to 12 trustees, Joshua Bayes signing as clerk. Urwick also notes that Joshua probably remained in Box Lane for some 11 years before removing to Southwark. Thus Thomas was probably born in Bovingdon.

The name of Elias Wordsworth, mentioned in Bayes' will, raises a problem. Clay (1894–1895) recorded, in the Wordsworth stemma, an Elias Wordsworth (Elias I, let us say), who was married to Ruth Bayes (daughter of Thomas' paternal grandfather Joshua) and who died in 1723. This couple had a son, also Elias (II), born in 1695, who died in 1740. This would surely be the cousin named in Thomas' will—but would we not then expect Thomas to have known of the death and so not to have mentioned him in his will? *However*, Clay also recorded that, on Ruth's dying, Elias I married Ann Milner, and even though he writes that Ann "has £12 a year by her son in law Elias' will; lived with her son Elias," it appears from the details he gives that "her son Elias" is not Elias III but Elias II. (Note: "Son-in-law" is an obsolete form of our "step-son": cf. Dickens' use of "mother-in-law" in *The Pickwick Papers*.)

Was Thomas Cotton *fils* a shady character? Joshua Bayes left £1400 in his will to his daughter Rebecca, who had married Thomas Cotton. However, in a codicil this bequest was revoked and she was left only £40 for mourning. (The original bequest was left in Trust "so that the same may not be subject to [Thomas Cotton's] debt," due arrangements being made for its disimbursement in the event of Cotton's death.)

While the issue of the Elias Wordsworth mentioned in Bayes' will raises a slight problem, the mention in that document of one Sarah Jeffrey, daughter of John Jeffrey, is even more awkward. One possible candidate is certainly the Sarah mentioned in the *International Genealogical Index* as having been baptized on the 29th January 1724. Roger Farthing, historian of Tunbridge Wells, however, finds this "Sarah-IGI" an unlikely candidate, in view of the fact that her mother and father were married on the 1st January 1728, and he thinks it unlikely that there would have been a birth before the nuptials had been celebrated. The Sarah of Bayes' will, say Sarah-T, was unmarried in 1760 when the will was drawn up, but in her father's will, drawn up in 1769, she is referred to as married. Sarah-IGI is known to have married her first husband, the wheelwright Robert Jeffery, in 1762 and to have had four children. I myself am prepared to accept that Sarah-IGI $\equiv$ Sarah-T—*pace* Farthing.

TUNBRIDGE WELLS

As Bellhouse has noted, we do not know when Bayes moved from London to Tunbridge Wells. He was appointed, as assistant to his father, at the Leather Lane Chapel in 1728 and was certainly attached to the Mount Sion Chapel in the early 1730s. It was, however, the custom for ministers to be invited to preach at the latter chapel, and Bayes could well have been one of these guest preachers before his permanent appointment.

At least two origins have been proposed for the names *Mount Sion* and *Mount Ephraim*. According to one of these, the hills were named after a fancied (or fanciful) resemblence of the site to Jerusalem, while another traces *Mount Sion* to an innkeeper who thus named his hostelry.

While Whiston recorded not only his breakfast meeting with Bayes on St. Bartholomew's Day in 1746, but also one of their topics of conversation on that occasion, there has been speculation as to the reason for the meeting. I have elsewhere suggested three possibilities: (a) both Whiston and Bayes were dissenters who shared an interest in scientific matters, (b) Humphrey Ditton, a predecessor of Bayes in the Mount Sion Chapel, had collaborated with Whiston and (c) on removing to London from Cambridge in the early 18th century, Whiston lived in Cross Street, very near to the Bayes family home in Little Kirby Street.

Comment

A. W. F. Edwards

Dedicated to the memory of G. A. Barnard (1915–2002), first biographer of Bayes

INTRODUCTION

Thomas Bayes was one of R. A. Fisher's heroes. When Fisher was President of the Royal Statistical Society in 1953, he campaigned for the inclusion of Bayes in the (British) *Dictionary of National Biography* (*DNB*). Writing on behalf of the Council, he remarked (Bennett, 1990):

> [I]n the twentieth century [Bayes] has become one of the best known figures in the history of the development of our understanding of inductive reasoning.... Biographies already exist in your *Dictionary* of his father and grandfather who were distinguished as dissenting Ministers, but not of their descendant who was Fellow of the Royal Society for twenty years, and has gained perhaps a more lasting celebrity.... My Council would all be gratified if, through their mediation, it were possible to add to the national series a biographical notice worthy of so remarkable a subject.

However, the *DNB* was not in the habit of repairing omissions, until 1989, when it decided to canvass opinion for names to include in a *Missing Persons* volume. Many Fellows of the Royal Statistical Society nominated Bayes, and perhaps because I was the straw which broke the camel's back, I was landed with the task of writing his entry (Edwards, 1993). In truth it was a joint effort, for I consulted all the best authorities, including D. R. Bellhouse. How much has been discovered since then! Bellhouse has done science and mathematics a great service through his Bayesian studies, now gathered together with those of others, especially A. I. Dale, into his *Biography*.

In fact the very word "Bayesian" was coined by Fisher in 1950—but more of that anon. First we may note that, in 1953, Fisher had appointed G. A. Barnard as one of his Vice Presidents, and he mentioned to the *DNB* that Barnard had been gathering particulars of Bayes' family and career. These particulars matured into the first biography of Bayes to be published (with thanks to Fisher "for some initial prodding which set him moving"), accompanied by a reprinting of Bayes' famous paper (Barnard, 1958).

WHEN DID FISHER FIRST READ BAYES?

In 1936, in his address at the Harvard Tercentenary Conference, Fisher (1936) remarked of the theory of inverse probability, "I may myself say that I learned it at school as an integral part of the subject, and for some years saw no reason to question its validity." One also sees from this address that in 1936 Fisher had been reading Laplace's reference to Bayes in his *Essai philosophique sur les probabilités* in the second edition of *Théorie analytique des probabilités* (Laplace, 1820) as well as De Morgan's remark (1838) that the "inverse method ... was first used by the Rev. T. Bayes, in *Phil. Trans*. liii. 370.; and the author, though now almost forgotten, deserves the most honourable remembrance from all who treat the history of this science." Fisher quoted this passage from De Morgan in his letter to the *DNB*.

"At school," of course, Bayes' name itself might not have been mentioned. It is also absent from the statistical texts with which Fisher was probably familiar as a student. Aldrich (1997) and Edwards (1997) have ransacked Fisher's early writings in attempts to trace his

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evolution of the method of maximum likelihood and of his attitude to inverse probability (see also the introduction to these two articles by Fienberg, 1997). Confusion reigned until 1921 when, in Aldrich's words, "Fisher quietly uncoupled the absolute criterion [maximum likelihood] from the method of inverse probability."

Fisher (1921) was the article in which he first mentioned the name of Bayes and referred explicitly to his work; it was a response to Soper, Young, Cave, Lee and Pearson (1917). It is also the article which prompted Fisher to coin the word "Bayesian" in 1950, when he wrote a short introduction to it for his *Contributions to Mathematical Statistics* (Fisher, 1950), referring to "Bayesian probabilities [of hypotheses] *a posteriori*." He repeated the phrase in the introduction to another article, "Inverse Probability," in the same volume (Fisher, 1930).

(This story has often been told in recent years; see Aldrich, 1997, and Edwards, 1997, and references therein.) Soper et al. (1917) thought that Fisher had used "Bayes' theorem" in 1915 (Fisher, 1915). They did not mince their words:

> Bayes' Theorem must be based on experience, the experience that where we are *à priori* in ignorance all values are equally likely to occur. . . . the indiscriminate use of Bayes' Theorem is to be deprecated. It has unfortunately been made into a fetish by certain purely mathematical writers on the theory of probability, who have not adequately appreciated the limits of Edgeworth's justification of the theorem by appeal to *general* experience.

A DIGRESSION: WHAT IS "BAYES' THEOREM"?

Nowadays "Bayes' theorem" usually means a simple and uncontroversial theorem in conditional probabilities, but when the phrase was coined by Lubbock and Drinkwater in 1830 (David, 2000) and used by Todhunter (1865), it meant the whole inverse argument in the binomial case according to which (as we should now say) the posterior probability for a binomial parameter might be obtained from the likelihood by assuming a uniform prior probability distribution. It thus fully incorporated the postulate in Bayes' own (1763) *scholium*. This meaning lasted nearly a century: Keynes (1921) could still write "Bayes' Theorem is the Inverse Principle of Probability Itself." However, already in 1922, Fisher noted some confusion:

> The [binomial] *result*, the *datum* [uniform prior in the particular binomial case], and the *postulate* implied by the *scholium*, have all been somewhat loosely spoken of as Bayes' Theorem.

[A quotation duly recorded in the *Oxford English Dictionary* definition. A recent book (Howie, 2002) makes the same point.]

The use of the phrase to refer to the simple theorem in conditional probabilities seems to have started with mathematicians writing texts on probability. Of books that come readily to hand, I find Coolidge (1925) and Levy and Roth (1936) regard the theorem as one in conditional probability alone: Coolidge called it "Bayes' Principle." Burnside (1928) judiciously refers to this as "Bayes' formula." After World War II, the usage was almost invariably the modern one (e.g., David, 1949; Good, 1950; Kendall, 1952; Neyman, 1950 did not mention Bayes at all). Savage (1954) calls it "Bayes' rule (or theorem)." de Finetti (1974) notes "One must be careful not to confuse Bayes' theorem (which is a simple corollary of the theorem of compound probabilities) with Bayes' *postulate* (which assumes the uniform distribution as a representation of 'knowing nothing')." At the time I wrote *Likelihood* (Edwards, 1972) I was quite unaware of the original meaning.

Fisher's usage was conservative, however. The title of a 1926 article, "Bayes' Theorem and the Fourfold Table," referred to the original meaning (Fisher, 1926), and when he came to write *Statistical Methods and Scientific Inference* (Fisher, 1956) his usage was precisely the same as Todhunter's.

The change to the modern meaning engendered a parallel change in the use of the word "Bayesian," which nowadays often means any method that employs Bayes' formula. The medical literature especially is full of "Bayesian solutions" and "Bayesian approaches," which are no more than applications of conditional probability.

BACK TO FISHER (1921)

Fisher was 27 years old when Soper et al.'s (1917) criticism appeared. Stung into responding, he denied that he had used Bayes' theorem in 1915, but calmed sufficiently to add an appendix to his article (Fisher, 1921) titled "Note on the Confusion between Bayes' Rule and My Method of the Evaluation of the Optimum," in which he coined the word "likelihood" in the

course of distancing his method of estimation from that of maximizing a Bayesian posterior distribution.

From 1921 onward, Fisher campaigned against what he saw as the unwarranted use of Bayes' theorem (original sense). In this he was, of course, influenced by the advances he was simultaneously making in mathematical and statistical genetics, where parameters under estimation, notably gene frequencies, may well have "prior" distributions arising from the sampling of populations, justifying the use of the theorem (Plackett, 1989). More than anything else, Fisher's book *Statistical Methods for Research Workers* (1925) spearheaded the exclusion of Bayesian methods from respectable statistical practice for the next 50 years. In the first edition, he explicitly stated his "personal conviction ... that the theory of inverse probability is founded upon an error, and must be wholly rejected."

Fisher, the mathematician, admired Bayes for his mathematical exposition. Fisher, the philosopher of induction, admired him for his alleged caution (Fisher, 1934):

> To Thomas Bayes must be given the credit of broaching the problem of using the concepts of mathematical probability in discussing problems of inductive inference, in which we argue from the particular to the general; ... Bayes put forward, with considerable caution, a method by which such problems could be reduced to the form of problems of probability.
>
> To the merit of broaching a fundamentally important problem, Bayes added that of perceiving, much more clearly than some of his followers have done, the logical weakness of the form of solution he put forward. Indeed we are told that it was his doubts respecting the validity of the postulate needed for establishing the method of inverse probability that led to his withholding his entire treatise from publication. Actually it was not published until after his death.

Stigler (1982, 1986) challenged this view, I think correctly. He notes, as had Molina (1930) and I (Edwards, 1978), that Bayes' *scholium* contained a cogent argument for adopting a uniform prior distribution in the particular binomial case that he was considering (though it does rather beg the question). This point had not only escaped Fisher, but Karl Pearson, Harold Jeffreys, D. V. Lindley and Ian Hacking as well. Indeed, one of Fisher's repeated criticisms of the adoption of a uniform prior was that it was not invariant to parameter transformation. Additionally, it is not clear on what evidence Fisher believed that Bayes withheld publication deliberately.

Nevertheless, as Stigler (1982) emphasized, Bayes was referring only to the binomial case. The argument of the *scholium* is specific to that case and lends no support to the general adoption of inverse probability. It conforms to the view that only in the context of betting is one obliged to behave as if one were a Bayesian (in the fullest sense). Fisher can admire Bayes but reject inverse probability as a panacea for induction without a charge of inconsistency.

Extensive further commentary on Bayes' paper is provided by Gillies (1987), Hald (1998) and Dale (1999).

FISHER'S 1956 COMMENTARY ON BAYES' ARTICLE

Zabell (1989) has written a persuasive critique of Fisher's history of the decline in the use of inverse probability contained in *Statistical Methods and Scientific Inference* (Fisher, 1956). In particular, Fisher's comments on Boole, Venn and Chrystal reveal a rather shallow knowledge of those writers. It is interesting that Fisher's first references to them were in 1922 (Fisher, 1922), soon after he had been provoked into denying that his method of maximum likelihood relied on inverse probability, and he no doubt found comfort for his point of view in the extracts on which he lighted. Since no one seems to have called into question his comments on Boole, Venn and Chrystal between 1922 and 1956, Fisher might be forgiven for failing to modify his early opinion (during a period in which his own age had doubled!). Only in recent decades has the subject of the history of probability and statistics achieved the rigor of a proper academic discipline.

In *Statistical Methods and Scientific Inference*, prior to these observations on 19th century writers, Fisher included a commentary on Bayes' own article. He repeated his speculation that Bayes withheld publication because of his doubts about the postulate. He also repeated his criticism of Bayes for not recognizing that his argument was not invariant to transformations of the parameter, but, as we have seen, this was to overlook the argument of the *scholium*. It is unfortunate that in preparing this commentary, Fisher seems not to have reexamined the *scholium* after an interval of 34 years, for in 1922 he had at least noted:

> After giving this solution ... Bayes adds a *scholium*, the purport of which would seem to be that in the absence of all knowledge save that supplied by the sample, it is reasonable to assume this particular *a priori* distribution of p.

Molina (1930) noted that Fisher here overlooked the full significance of the *scholium*. In other respects, Fisher's commentary seems satisfactory.

In addition to this commentary Fisher (1956) constructed an ingenious argument for a particular uninformative prior distribution in the binomial case, $p^{-1/2}(1-p)^{-1/2}\,dp/\pi$. From a certain point of view, the fiducial argument provides a justification for the Jeffreys priors in examples of location-and-scale distributions, and although the discreteness of the binomial distribution renders an exact fiducial argument impossible, Fisher produced an approximate argument. Both the first (Fisher, 1956) and the third, posthumous (1973, 1990), editions of his book should be consulted for details. Perhaps unsurprisingly, the above prior distribution is the one which results from giving ϕ a uniform distribution when $\phi = \sin^2 p$, Fisher's variance-stabilizing angular transformation for the binomial. Fisher noted this fact, but did not connect it to his fiducial justification. However, ϕ, by virtue of possessing a constant variance to first order, has become approximately a location parameter, to which the fiducial argument applies. Jeffreys (see his 1961) had derived this prior (often called after him) using an invariance argument based on Fisher information.

One way to look at Bayes' derivation of the uniform prior in the *scholium* is almost fiducial: the data, imagined to consist of a uniform distribution of the number of successes in n trials, induce a distribution for p, the parameter. It is doubly regrettable that Fisher did not take the point of the *scholium*; had he done so, he would have had to contemplate the difference between the two priors. The general argument, however, is the same as the argument for the Jeffreys priors in the case of location-and-scale distributions: they are the priors that deliver the accepted repeated-sampling results because they lead to the fiducial distributions for the parameters.

Barnard (1987) wrote a paper titled "R. A. Fisher–A True Bayesian?," which takes this story one step further. The last sentence of the summary reads, "Fisher was not a 'Bayesian' in the main current sense of the word." Rather, Bayes was a paleo-fiducialist, because he argued that the data could induce a probability distribution on a parameter that summarized the information about it but which was not itself a frequency distribution of its occurrence in the real world. Jeffreys came closest to seeing this connection, and had de Finetti been as familiar as was Jeffreys with the fiducial argument, he would have seen it too, but in fact his insight is confined to a footnote (de Finetti, 1975):

> It seems to me that he [Fisher] felt the need for the Bayesian form of conclusion (although he expressed it in an illusory manner by means of an undefinable 'fiducial probability'), but wanted to approach the problem from the opposite direction (an approach rather like that of Neyman).

NOTE ON REPRINTS OF BAYES' ARTICLE

Now that *Philosophical Transactions* is available electronically on JSTOR, reprints of Bayes' paper are only of historical interest, but important for the study of its slow acceptance. In their 1809 abridgement of *Philosophical Transactions*, Hutton, Shaw and Pearson declined to include it ("In its full extent and perfect mathematical solution, this problem is much too long and intricate, to be at all materially and practically useful, and such as to authorize the reprinting it here"), but Dale (1999) noted an immediate (1764) reprint in Edinburgh University Library. There was also the *Philosophical Transactions* reprint in 1774 in Wittenberg, Germany, by C. C. Dürr (drawn to my attention by my brother, J. H. Edwards, and identified by S. M. Stigler), and another in *The Works of Dr Richard Price* (1816). It seems there was then a gap until the 1940 facsimile reprint with a commentary by E. C. Molina, coupled with a reprint of Bayes' letter to John Canton on asymptotic series with a commentary by W. Edwards Deming, whose purpose was "to lift the essay from the obscurity of a few extant volumes of the Transactions of the Royal Society" (cited in my references as Molina and Deming, 1940). That obscurity had, of course, been assisted in some respects by Todhunter's *History* (1865) from which so many then derived their knowledge of the essay.

It was the reprinting by Barnard (1958) in the admirable *Biometrika* series "Studies in the History of Probability and Statistics," coupled with the inclusion of Barnard's paper in Pearson and Kendall (1970) that made Bayes' paper so widely available. Dale (1991a) lists translations into other languages.

Comment

D. V. Lindley

An intellectual game I used to play involved discussing whom you would like to meet when you reached Heaven, assuming you were so fortunate, and what you would say to them. "Shakespeare, Sir? Sorry, his waiting list is so long that it has had to be closed. Perhaps the Earl of Oxford?" Were I to play this game nowadays it would seem sensible to ask for Thomas Bayes, with presumably a short waiting list, but I wonder if the conversation would be rewarding. (An imagined one was presented at one of the Valencia meetings on Bayesian statistics.) Did he really prove the theorem and write the article supposedly found by Price? David Bellhouse has done the statistical community a great service in producing a very fine biography, discovering more interesting and important facts than I had anticipated being available, and presenting them in such a clear and elegant manner. The contrast between our understanding of Bayes, the man, now and when I first became interested in him through Harold Jeffreys' lectures in 1946, is very great indeed thanks to the work of Bellhouse and Dale. However, we do not seem to be much nearer to understanding how he became interested in the problem solved in the article. Mathematics is a young person's activity and therefore we might expect that Bayes was at his mathematical best around 1725 when, unfortunately, there is a gap in the record after his departure from Edinburgh and no mention of his having mathematical contacts. Around that time, de Moivre, cognizant of the binomial result concerning r successes in n independent trials with a constant probability p of success on each trial, and working on the limiting result, might well have mentioned the inverse problem concerning p on the evidence provided by r and n, the problem solved by our hero. Did the solution to the inverse problem come around then and is Bayes "the ingenious friend" mentioned in 1749, or is it Saunderson, as Stigler (1983) suggested in his entertaining, yet learned, article. If it did, why did the author delay publication? When Bayes' paper did appear, why did the event pass unnoticed? Bellhouse suggests that Bayes might first have become interested in probability as late as 1755, but, if so, it is unexpected that he should have made such a major, original advance at his mature age, especially when he was not well. Against this, he had the leisure to pursue mathematical interests and the late flowering of his talent could account for the article occurring only at his death. We shall likely never know and the heavenly conversation could therefore be worth having.

According to Stigler's (1980) law of eponymy, no discovery is named after its first discoverer, so perhaps we should question whether Bayesian statistics is appropriately named, even if he was a discoverer. My opinion is that it is not. We do admittedly make extensive use of Bayes' theorem in essentially the same way as in the article, but surely there is more to it than that. For me, the defining quality of our subject is the recognition that probability is the only sensible description of uncertainty. More correctly, statements of uncertainty must combine according to the rules of the probability calculus. Emphasis in that last sentence is on the word "must" and in the previous one on "only." Even frequentists use probability, but also employ additional concepts like confidence to describe uncertainty, in violation of the probability calculus. Bayesians in the 19th century used probability, but failed to justify their use of it. It is only in the 20th century that we have proofs of the inevitability of probability, the earliest of them being due to Frank Ramsey and to Bruno de Finetti. If I had to choose between them, my choice would fall on Ramsey, because his argument embraced decision analysis, whereas de Finetti's did not. Subsequent workers in Bayesian statistics have followed the ideas laid out by de Finetti, rather than Ramsey, but part of the reason for this might be that Ramsey died so young and did not have the chance to develop his original ideas, whereas de Finetti did. Certainly today a student would get more from pursuing de Finetti, than Ramsey's small output. On the other hand, to transfer statistical concepts into action, one needs more; the notion of utility and that of maximizing expected utility, both of which Ramsey used, also recognizing the intimate connection between utility and probability.

So in the game, I would ask to meet with that genius who died at the age of 26 after having made important contributions to philosophy and economics in addition to our own field. Did anyone recognize

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his centenary on 22 February 2003? I opened a bottle of Burgundy in his honor. Frank's younger brother, who physically resembled him, was later to become Archbishop of Canterbury, the head of the Anglican Church, and I often used to hear this giant of a man on the radio with his strange, compelling mode of speech, and wish that I was listening instead to his brother talk about inference rather than the religious topics of the Archbishop. There is a brief biography of Ramsey by Newman (1990) and there is a moving account in Partridge (1981) of his death, just after he had been visited by Wittgenstein.

Some people will sensibly suggest that the proofs of de Finetti and Ramsey were not sufficiently rigorous and that the true discoverer is L. J. Savage. Not only did he provide a splendid proof, but he was among the first to appreciate the power of the approaches adopted by the Italian and the Englishman, and make them understandable to the rest of us less brilliant minds. However, Savage statistics does not sound well. Supported by the law of eponymy, I do not approve of naming scientific advances after individuals, even though I did name one after Cromwell; only later did I realize I was wrong, since it was already an integral part of Jain philosophy.

Nevertheless, a name is needed for what we currently call Bayesian statistics. Both Jeffreys and de Finetti called it simply probability and this is logical if my assessment above of the key idea behind it is correct. However, the word is already used by people for a topic that has no inferential content. Another possibility is to call the subject coherent statistics, for coherence is a central concept in what Bayesians do; a coherence achieved through the use of the probability calculus. It has the attractive consequence that concepts like tail-area significance tests and confidence intervals belong to incoherent statistics. So perhaps, rather than ask to see Bayes when at the gates of Heaven, one should ask St. Peter whether they are coherent up there. They must be mustn't they?

Comment

Stephen M. Stigler

Thomas Bayes has been an enigmatic figure of the history of our science. His work and his life were paid little attention in his own time, and only in the 20th century did he become a figure of more than passing and superficial interest. The change dates to the rise in interest in inverse probability in the works of Edgeworth, Jeffreys, Savage, de Finetti and others, the reprinting of Bayes' "Essay" by W. Edwards Deming in 1940, and the appearance of George Barnard's life of Bayes in 1958. All sources, down to and including David Bellhouse's biography (which I expect will be the definitive story for many years), omit one of the most basic facts about Bayes: his birthday.

Birth dates are the smallest of biographical facts, and one might think they should be of interest only to astrologers and followers of those dark arts. If that were the case, why is so much attention paid to them in society, and even by the scholarly community? Birthdays are celebrated in every family, and for major scholars they can be the occasions for conferences or festschrifts. Probabilists have grown accustomed to invigorating the classroom with a demonstration of the Birthday Problem. Compilations have been published in many fields (e.g., Stigler, 1981; Stigler and Friedland, 1989; Blackburn and Holford-Strevens, 1999, is an invaluable auxiliary reference). Their implications have been examined for disciplinary history (Stigler, 2002). Articles and chapters have been written on a single date (e.g., Barner, 2001; Stigler, 1999, Chap. 13). Who above the age of six and in sound mind does not know their own birthday or has not at least adopted *some* calendar date as its surrogate? Of what other species of biographical fact is there as wide general knowledge? Clearly birthdays are important, even if only as excuses for celebration. But, when should we celebrate Thomas Bayes?

Accurate and complete biography depends on contemporary records, and in Bayes' case the records are skimpy to the extreme. Essentially, all we have is the statement carved on Bayes' tomb, that he died April 7, 1761 at age 59. From this, even the year is not known. Bellhouse writes "all that can be said about Bayes'

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birth date is that it is probably between July of 1701 and April of 1702." Bayes was a first child, and Bellhouse evidently picks the July limit, rather than April 1701, because that is nine months subsequent to the date he has found for Bayes' parents' marriage (the marriage license was issued October 23, 1700, and the marriage was probably then or very soon after). Now, this is a genteel gesture on his part, but it is contradicted by extensive research by historical demographers, who find from parish records that circa 1700 in England about a quarter of the marriages involved pregnant brides. There is even very limited data from a Nonconforming parish that gives a slightly higher figure, 3/8. There was in those days a stigma attached to birth out of wedlock, but not to pregnancy at marriage (Hair, 1966, 1970; Wrigley and Schofield, 1981, pages 254–255, 366–368).

To get a better fix on things, I have obtained data from the NORC (2001) National Longitudinal Survey of Youth for women born 1957–1964 in the United States (Figure 1). These data give the time in months from marriage to first birth for those whose first birth occurred after marriage. About the same percentage of prenuptial pregnancies (pregnant brides) occurred in this sample as is found in the parish records of Bayes' time, giving confidence in the relevance of these data to Bayes' situation. From these data, conditioning on Bayes' death date, his age at death, and the date of his parents' marriage license, and using Bayes' theorem, I find the posterior probability that Bayes was indeed born in the year 1701 is 0.81, just over 80%. Boldly carrying the matter further, I have computed the posterior expectation of his birth date to be September 10, 1701. This would indicate that

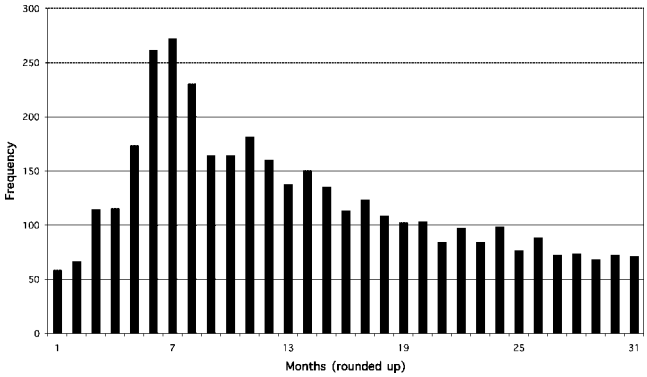

FIG. 1. *Number of months from marriage to first birth* (*NLSY79, data 1982–1998*).

Bayes' mother was not a pregnant bride, although the single most likely month for his birth, the month from April 7 to May 6, 1701, would imply otherwise. I did this analysis by treating the empirical histogram from the National Longitudinal Survey as providing the unconditional distribution of birth interval after marriage, and assumed the distribution within months was uniform. I also took the marriage license date of October 23, 1700, as the marriage date, and I accepted the statement on Bayes' tomb as accurate. Of course the result is only approximate, but then so are most early recorded birth dates. If you wish more detail, I could add that the birth is estimated to have occurred in the afternoon at 4:09 PM, but I would not place much faith in that time.

Bayes was a minor figure in his own time, but an icon to our age. I suggest that each year on September 10, we all raise a toast to this remarkable historical figure and to the dedicated scholars who have so greatly advanced our knowledge of Bayes.

Rejoinder

David Bellhouse

First, I would like to thank the discussants for their interesting and very informative comments. There is one other person I need to thank and that is George Styan. Many years ago Professor Styan ran a contest in *The IMS Bulletin*. There was a picture of a clergyman with two questions posed about him: Who was the man and when and where was he born? The picture is supposed to be of Thomas Bayes, but there are doubts as to the authenticity of the portrait, doubts I have noted in my article. I never did find his birth date; Professor Stigler in his discussion provides the best answer I have ever seen. The upshot of this contest was that I am still not sure that I correctly provided the true identity of the person in the picture and I never successfully found Bayes' birth date. At the time, I won the prize for the best answer, a copy of *The History of Statistics* (Stigler,

1986), and I developed a passion for finding out more about Thomas Bayes.

Professor Dale has made some excellent points regarding Bayes' religious beliefs and his family background. The only points I want to address from his discussion are the places where Thomas Bayes' father, Joshua, and great-uncle, Samuel, may have lived and ministered. I agree that there is room for doubt; I would like to try to strengthen the arguments in favor of London and Manchester, respectively. My argument is based on Benjamin Mills' funeral sermon for John Archer, who preceded Bayes as minister at Tunbridge Wells (Mills, 1733). Mills said that he and others had regularly traveled from London to Tunbridge Wells to take services for Archer while he was sick. The distance from London to Tunbridge Wells is 36 miles.

Dr. Williams Library in London, England, has a card index with dates regarding the pastoral careers of various Nonconformist ministers. The index entry for Joshua Bayes shows the indexer's best approximation to his career: Box Lane, Hertfordshire, 1694–1706, St. Thomas' Southwark, assistant, 1706–1723, Leather Lane, assistant 1697(?)–1723 and Leather Lane, pastor, 1723–1746. Box Lane is in Hemel Hempstead, a market town near Bovingdon. The town is 24 miles from London, two-thirds the distance that Mills needed to travel to take over Archer's Sunday service in Tunbridge Wells. The *Dictionary of National Biography* entry (Stephen and Lee, 1921–1922) for Joshua Bayes states, "It appears that young Bayes 'served' the churches around London as a kind of itinerant or evangelist for some years." It is possible that around 1700, Bayes was living in London and visiting his churches by horseback. A minister's main duty was taking a service on Sunday and preaching the sermon; it was not necessary to be present in the town during the week. The same may be true for Samuel Bayes. Sankey is a town 2 miles from Warrington. Warrington, halfway between Manchester and Liverpool, is 18 miles west of Manchester. Again, Samuel Bayes could have served Sankey from Manchester by horseback. It is also possible that he could have taken some river conveyance down the Irwell and Mersey Rivers; Warrington is on the Mersey and the Irwell, part of the Mersey water system, flows through Manchester. Likewise, travel to Hemel Hempstead may have been possible by water. The Gade River flows through Hemel Hempstead and into the Colne River, which flows into the Thames.

Professor Lindley has commented that "Mathematics is a young person's activity ..." and, consequently, Bayes' initial interest in probability dating from 1755 is unlikely, so an earlier date might be considered. There is truth to the truism, but there are also exceptions. Consider de Moivre. Born in 1667, he came to England in about 1688 (Stigler, 1986) and began publishing mathematical articles in 1695 (de Moivre, 1695). His career was "held up" as he tried to establish himself in a foreign country. His most influential work began appearing in 1711 (de Moivre, 1711) at the age of 44. This was his initial article on the subject of probability and there is no evidence that he had any interest in the subject prior to that time. The topic was suggested to him by Francis Robartes, later the Earl of Radnor, to whom de Moivre dedicated the article.

Like Professor Lindley I have always been curious about Bayes' disinterest in the publication of his own work. This curiosity became agonizingly more unsatisfied when I discovered Bayes' manuscripts in the Stanhope collection. Bayes had found the correct derivation of Stirling's approximation to factorials as early as 1747, but had never bothered to publish his result. The result was circulated among his friends, including Canton and Stanhope in particular, so it must have been a result that he felt was of some importance. The result appeared in print posthumously (Bayes, 1763b); and I have this nagging suspicion that Bayes' request to Canton, which appeared in the opening sentence of the article, to have the result published was actually written by Canton. Among Bayes' sponsors to the Royal Society, all but one had publications. The one exception was Stanhope. As indicated in Murdoch's letter to Stanhope concerning the third edition of de Moivre's *Doctrine of Chances*, Stanhope also may have had publishable, but never published, mathematical results.

Regarding Professor Stigler's Bayesian analysis of Bayes' birth date, I tried to add some refinements of my own, but was unsuccessful. Initially, I thought that one should condition on the relevant subset of the data, the fact that Joshua Bayes was a Presbyterian minister. That might shift the birth date to a few days or weeks after September 10. It occurred to me, however, that I was also imposing 20th century Presbyterian values onto perhaps a different kind of Presbyterian from the very early 18th century. Then I discovered an article by Laslett and Oosterveen (1973) that showed that the illegitimacy rate in the decades around 1700 was low in comparison to the 19th and 20th centuries, so in compensation, perhaps bridal pregnancy was on the rise at the time, thus resulting in an earlier birth date. In the end I could find no appropriate refinement. I very much like Professor Stigler's result for both Bayesian

and non-Bayesian reasons. On the non-Bayesian side, September 10 is just after my university term begins and can therefore be celebrated appropriately in the company of colleagues without the distractions that come closer to the time when the first term assignment or test is due. An additional benefit is that 4:09 in the afternoon seems to be a very good time to begin the celebration.

ADDITIONAL REFERENCES

ALDRICH, J. (1997). R. A. Fisher and the making of maximum likelihood 1912–1922. *Statist. Sci.* **12** 162–176.

BARNARD, G. A. (1987). R. A. Fisher—A true Bayesian? *Internat. Statist. Rev.* **55** 183–189.

BARNER, K. (2001). How old did Fermat become? *NTM (N.S.)* **9** 209–228.

BENNETT, J. H., ed. (1990). *Statistical Inference and Analysis*: *Selected Correspondence of R. A. Fisher*. Clarendon, Oxford.

BLACKBURN, B. and HOLFORD-STREVENS, L. (1999). *The Oxford Companion to the Year*. Oxford Univ. Press.

BURNSIDE, W. (1928). *Theory of Probability*. Cambridge Univ. Press. (Reprinted in 1959, Dover, New York.)

COOLIDGE, J. L. (1925). *An Introduction to Mathematical Probability*. Oxford Univ. Press. (Reprinted in 1962, Dover, New York.)

DALE, A. I. (1999). *A History of Inverse Probability*, 2nd ed. Springer, New York.

DAVID, F. N. (1949). *Probability Theory for Statistical Methods*. Cambridge Univ. Press.

DAVID, H. A. (2000). First (?) occurrence of common terms in statistics and probability. In *Annotated Readings in the History of Statistics* (H. A. David and A. W. F. Edwards, eds.). Springer, New York.

DAVID, H. A. and EDWARDS, A. W. F. (2000). *Annotated Readings in the History of Statistics*. Springer, New York.

DE FINETTI, B. (1974). *Theory of Probability* **1**. Wiley, London.

DE FINETTI, B. (1975). *Theory of Probability* **2**. Wiley, London.

DE MOIVRE, A. (1695). Specimina Quaedam Illustria Doctrinae Fluxionum Sive Exempla Quibus Methodi Istius Usus et Praestantia in Solvendis Problematis Geometricis Elucidatur, ex Epistola Peritissimi Mathematici D. Ab. de Moivre Desumpta. *Philosophical Transactions* **19** 52–57.

DE MOIVRE, A. (1711). De Mensura Sortis seu; de Probabilitate Eventuum in Ludis a Casu Fortuito Pendentibus. *Philosophical Transactions* **27** 213–264.

DE MORGAN, A. (1838). *An Essay on Probabilities and their Application to Life Contingencies and Insurance Offices*. Longman, Orme, Brown, Green and Longmans, London.

EDWARDS, A. W. F. (1972). *Likelihood*. Cambridge Univ. Press. (Reprinted in 1992, Johns Hopkins Univ. Press, Baltimore, MD.)

EDWARDS, A. W. F. (1978). Commentary on the arguments of Thomas Bayes. *Scand. J. Statist.* **5** 116–118.

EDWARDS, A. W. F. (1997). What did Fisher mean by "inverse probability" in 1912–1922? *Statist. Sci.* **12** 177–184.

FIENBERG, S. E. (1997). Introduction to R. A. Fisher on inverse probability and likelihood. *Statist. Sci.* **12** 161.

FISHER, R. A. (1915). Frequency distribution of the values of the correlation coefficient in samples from an indefinitely large population. *Biometrika* **10** 507–521.

FISHER, R. A. (1921). On the "probable error" of a coefficient of correlation deduced from a small sample. *Metron* **1** 3–32.

FISHER, R. A. (1922). On the mathematical foundations of theoretical statistics. *Philos. Trans. Roy. Soc. London Ser. A* **222** 309–368.

FISHER, R. A. (1925). *Statistical Methods for Research Workers*. Oliver and Boyd, Edinburgh.

FISHER, R. A. (1926). Bayes' theorem and the fourfold table. *Eugenics Review* **18** 32–33.

FISHER, R. A. (1930). Inverse probability. *Proc. Cambridge Philos. Soc.* **26** 528–535.

FISHER, R. A. (1934). Two new properties of mathematical likelihood. *Proc. Roy. Soc. London Ser. A* **144** 285–307.

FISHER, R. A. (1936). Uncertain inference. *Proc. American Academy of Arts and Sciences* **71** 245–258.

FISHER, R. A. (1950). *Contributions to Mathematical Statistics*. Wiley, New York.

FISHER, R. A. (1956). *Statistical Methods and Scientific Inference*. Oliver and Boyd, Edinburgh. [Reprinted (1990) *Statistical Methods, Experimental Design and Scientific Inference*. Oxford Univ. Press, London.]

GILLIES, D. A. (1987). Was Bayes a Bayesian? *Historia Math.* **14** 325–346.

GOOD, I. J. (1950). *Probability and the Weighing of Evidence*. Griffin, London.

HAIR, P. E. H. (1966). Bridal pregnancy in rural England in earlier centuries. *Population Studies* **20** 233–243.

HAIR, P. E. H. (1970). Bridal pregnancy in earlier rural England further examined. *Population Studies* **24** 59–70.

HALD, A. (1998). *A History of Mathematical Statistics from 1750 to 1930*. Wiley, New York.

HASTINGS, J., ed. (1971). *Encyclopædia of Religion and Ethics*, 12 vols. T. and T. Clarke, Edinburgh. (Reprint of edition published 1908–1921.)

HOWIE, D. (2002). *Interpreting Probability*. Cambridge Univ. Press.

HUTTON, C., SHAW, G. and PEARSON, R. (1809). *The Philosophical Transactions of the Royal Society of London, from their Commencement, in 1665, to the year 1800*; *Abridged, with Notes and Biographic Illustrations* **12** (from 1763 to 1769). Baldwin, London.

JEFFREYS, H. (1961). *Theory of Probability*, 3rd ed. Clarendon, Oxford.

KENDALL, M. G. (1952). *The Advanced Theory of Statistics* **1**, 5th ed. Griffin, London.

KEYNES, J. M. (1921). *A Treatise on Probability*. Macmillan, London.

LAPLACE, P. (1820). *Théorie analytique des probabilités*, 2nd ed. Courcier, Paris.

LASLETT, P. and OOSTERVEEN, K. (1973). Long-term trends in bastardy in England: A study of the illegitimacy figures in the parish registers and in the Reports of the Registrar General, 1561–1960. *Population Studies* **27** 255–286.

LEVY, H. and ROTH, L. (1936). *Elements of Probability*. Clarendon, Oxford.

MOLINA, E. C. (1930). The theory of probability: Some comments on Laplace's *Théorie analytique*. *Bull. Amer. Math. Soc.* **36** 369–392.

MOLINA, E. C. and DEMING, W. E. (1940). *Facsimiles of Two Papers by Bayes* (with commentaries). U.S. Department of Agriculture, Washington.

NEWMAN, P. (1990). Frank Plumpton Ramsey. In *Utility and Probability* (from *The New Palgrave*) (J. Eatwell, M. Milgate and P. Newman, eds.) 186–197. Macmillan, London.

NEYMAN, J. (1950). *First Course in Probability and Statistics*. Holt, New York.

NORC (2001). Data from the National Longitudinal Survey NLSY79. Data from 1982–1998. Courtesy of Rupa Datta and Parvati Krishnamurty.

PARTRIDGE, F. (1981). *Memories*. Victor Gollancz, London.

PEARSON, E. S. and KENDALL, M. G., eds. (1970). *Studies in the History of Statistics and Probability*. Griffin, London.

PLACKETT, R. L. (1989). Comment on "R. A. Fisher on the history of inverse probability," by S. Zabell. *Statist. Sci.* **4** 256–258.

PRICE, R. (1816). *The Works of Dr Richard Price with Memoirs of His Life by W. Morgan*, 10 vols. Rees, London.

SAVAGE, L. J. (1954). *The Foundations of Statistics*. Wiley, New York. Reprinted in 1972, Dover, New York.

SOPER, H. E., YOUNG, A. W., CAVE, B. M., LEE, A. and PEARSON, K. (1917). On the distribution of the correlation coefficient in small samples. Appendix II to the papers of 'Student' and R. A. Fisher. A cooperative study. *Biometrika* **11** 328–413.

STIGLER, G. J. and FRIEDLAND, C. (1989). *Chronicles of Economics Birthday Book.* Univ. Chicago Press.

STIGLER, S. M. (1980). Stigler's law of eponymy. *Trans. New York Academy Science, Series 2* **39** 147–158.

STIGLER, S. M. (1981). *Springer Statistics Calendar 1981*. Springer, New York. (Later, updated versions issued in 1983, 1985, 1986, 1987 and 1989.)

STIGLER, S. M. (1982). Thomas Bayes's Bayesian inference. *J. Roy. Statist. Soc. Ser. A* **145** 250–258.

STIGLER, S. M. (1983). Who discovered Bayes's theorem? *Amer. Statist.* **37** 290–296.

STIGLER, S. M. (1999). *Statistics on the Table*: *The History of Statistical Concepts and Methods.* Harvard Univ. Press, Cambridge, MA.

STIGLER, S. M. (2002). Statisticians and the history of economics. *J. History of Economic Thought* **24** 155–164.

TODHUNTER, I. (1865). *A History of the Mathematical Theory of Probability*. Macmillan, London. (Reprinted in 1949, Chelsea, New York.)

TURNER, G. L., ed. (1911). *Original Records of Early Nonconformity under Persecution and Indulgence*, 3 vols. Fisher Unwin, London.

URWICK, W. (1884). *Nonconformity in Herts. Being Lectures upon the Nonconforming Worthies of St. Albans, and Memorials of Puritanism and Nonconformity in All the Parishes of the County of Hertford*. Hazell, Watson and Viney, London.

WRIGLEY, E. A. and SCHOFIELD, R. S. (1981). *The Population History of England, 1541–1871*. Harvard Univ. Press, Cambridge, MA.

ZABELL, S. (1989). R. A. Fisher on the history of inverse probability (with discussion). *Statist. Sci.* **4** 247–263.